

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel
International GCSE**

Centre Number

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Candidate Number

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Time 2 hours

Paper
reference

4PM1/02

**Further Pure Mathematics
PAPER 2**



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ►

P66027RRA

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P 6 6 0 2 7 R R A 0 1 3 6



Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2} [2a + (n - 1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity, $S_\infty = \frac{a}{1 - r}$ $|r| < 1$

Binomial series

$(1 + x)^n = 1 + nx + \frac{n(n - 1)}{2!}x^2 + \dots + \frac{n(n - 1)\dots(n - r + 1)}{r!}x^r + \dots$ for $|x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$\tan \theta = \frac{\sin \theta}{\cos \theta}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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Question 1 continued

Handwriting practice area with 25 horizontal dotted lines.

(Total for Question 1 is 5 marks)

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- 2 The finite region enclosed by the curve with equation $y = 4 - x^2$ and the line with equation $y = x + 2$ is rotated through 360° about the x -axis.

Use algebraic integration to find the exact volume of the solid formed.

(6)

Area with horizontal dotted lines for writing the solution.

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Question 2 continued

Area with horizontal dotted lines for writing.

(Total for Question 2 is 6 marks)



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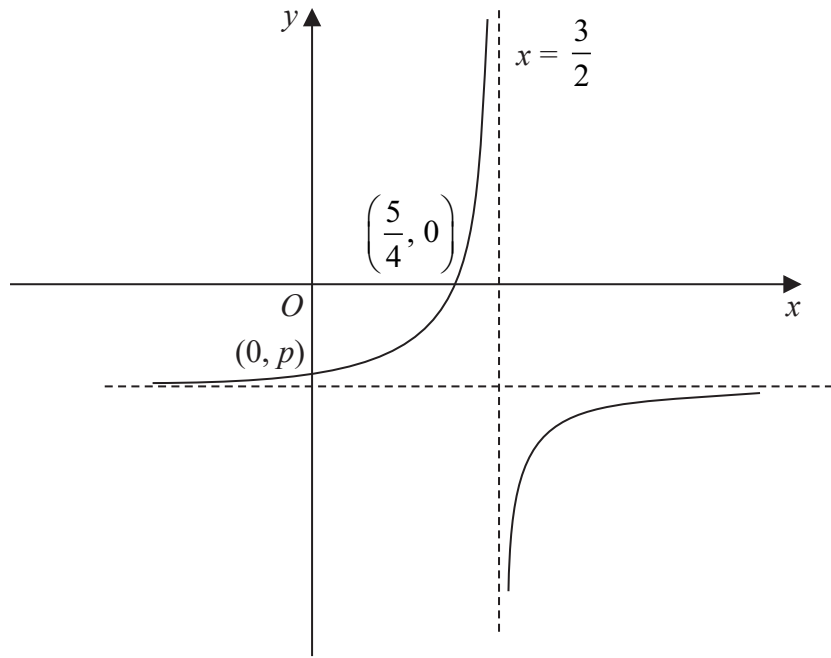


Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = \frac{a - bx}{cx - d} \quad x \neq \frac{d}{c}$$

where a , c and d are prime numbers and b is an integer.

The asymptote to the curve that is parallel to the y -axis has equation $x = \frac{3}{2}$

- (a) Write down the value of c and the value of d (2)

The curve crosses the x -axis at the point $\left(\frac{5}{4}, 0\right)$

- (b) Find the value of a and the value of b (2)

The curve crosses the y -axis at the point $(0, p)$ where p is a rational number.

- (c) Find the value of p (2)

- (d) Find an equation of the asymptote to the curve that is parallel to the x -axis. (1)

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Question 3 continued

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Question 3 continued

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Question 3 continued

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(Total for Question 3 is 7 marks)

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4 The curve C has equation $y = 2x^2 + px + q$ where p and q are integers.

The curve C has a stationary point at $(3, -5)$

(a) Show that $p = -12$ and find the value of q

(4)

(b) State, giving a reason, the nature of the stationary point.

(1)

(c) Find an equation of the normal to C at the point on C where $x = 1$

Give your answer in the form $ax + by + c = 0$

(6)

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Question 4 continued

Area for writing answers, consisting of multiple horizontal dotted lines.

(Total for Question 4 is 11 marks)

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Question 5 continued

Handwriting practice area with 25 horizontal dotted lines.

(Total for Question 5 is 6 marks)

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6 A particle P is moving along the x -axis. At time t seconds ($t \geq 0$) the displacement, s metres, of P from the origin O , is given by $s = t^3 - 4t^2 - 16t - 8$

(a) Find the distance of P from O when $t = 0$

(1)

(b) Find the value of t for which P is instantaneously at rest.

(4)

(c) Find the value of t for which P is accelerating at 10 m/s^2 in the positive x direction.

(3)

Dotted lines for writing answers.

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Question 6 continued

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(Total for Question 6 is 8 marks)

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7 A geometric series G has first term a and common ratio r

The sum of the first three terms of G is $\frac{61}{6}$

The sum to infinity of G is $\frac{125}{6}$

(a) (i) Show that $r = \frac{4}{5}$

(ii) Find the value of a

(6)

The sum of the first n terms of G is S_n

Given that $S_n > 19.8$

(b) show that $n \lg\left(\frac{4}{5}\right) < \lg\left(\frac{31}{625}\right)$

(2)

(c) Hence find the least value of n

(2)

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Question 7 continued

Handwriting practice area with 20 horizontal dotted lines.

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Question 7 continued

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Question 7 continued

Dotted lines for writing.

(Total for Question 7 is 10 marks)

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- 8 (a) Complete the table of values for $y = 2x + \frac{3}{x^2} - 3$ giving your answers to 2 decimal places where appropriate.

x	0.5	0.75	1	1.5	2	3	4	5
y	10		2				5.19	7.12

(2)

- (b) On the grid opposite, draw the graph of $y = 2x + \frac{3}{x^2} - 3$ for $0.5 \leq x \leq 5$

(2)

- (c) By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation $4x^3 - 10x^2 + 3 = 0$ for $0.5 \leq x \leq 5$

(5)

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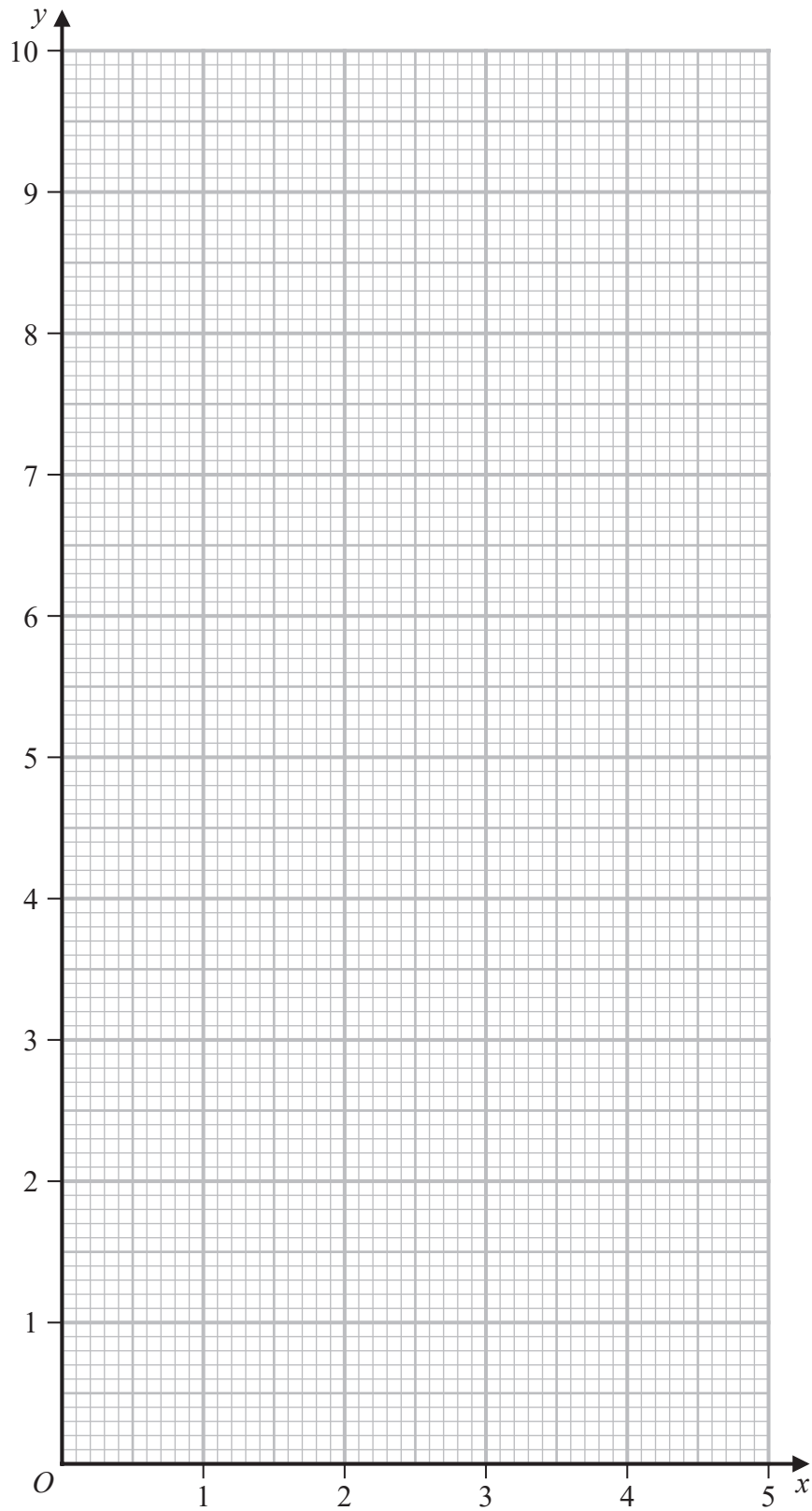
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Question 8 continued



Turn over for a spare grid if you need to redraw your graph.

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Question 8 continued

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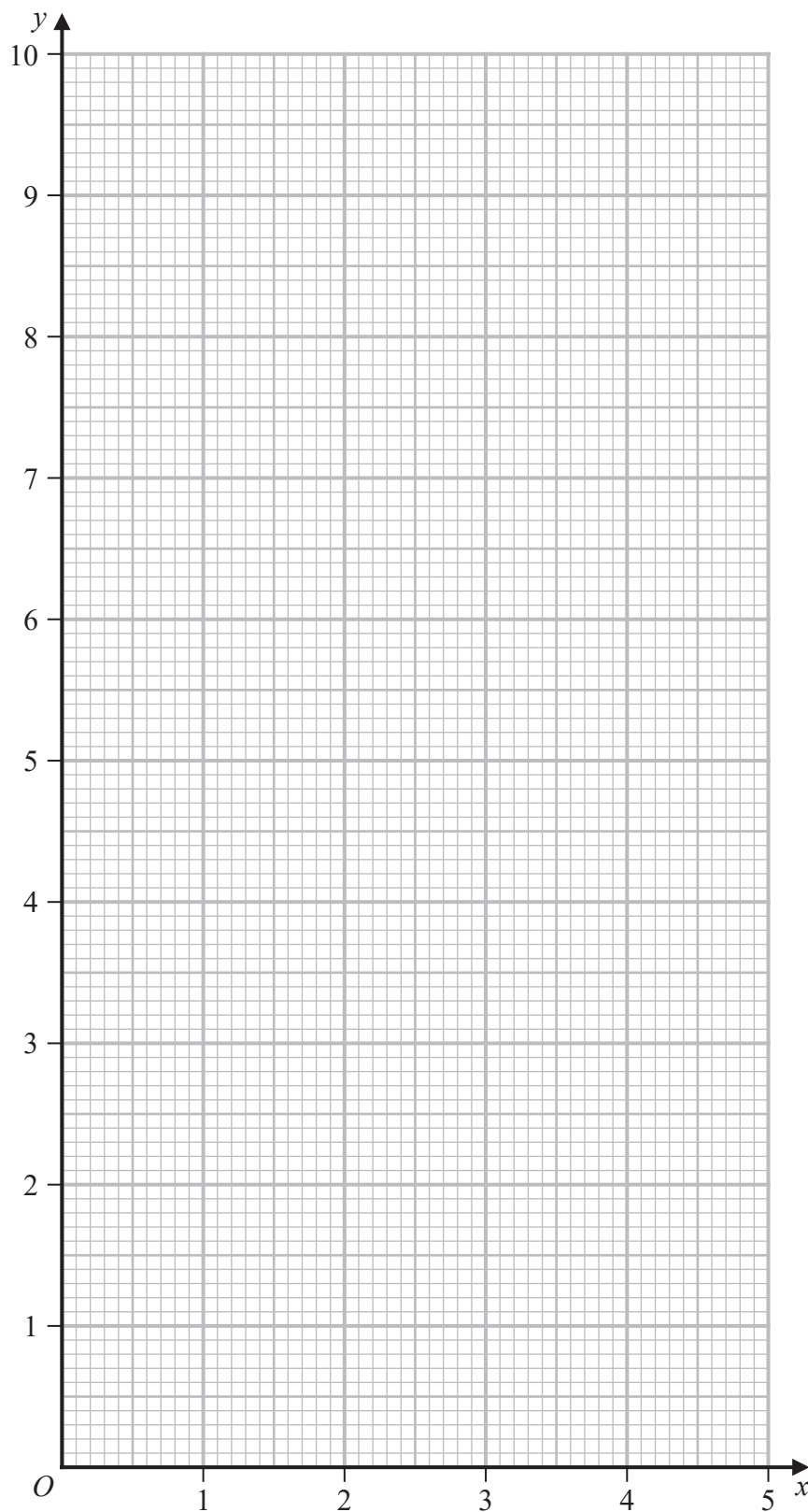
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Question 8 continued

Only use this grid if you need to redraw your graph.



(Total for Question 8 is 9 marks)



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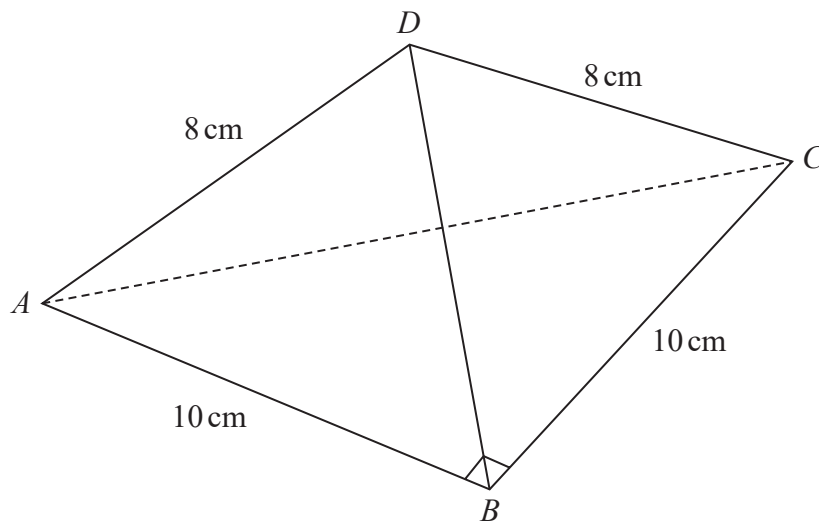


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a triangular pyramid $ABCD$ with base ABC

$$AB = BC = 10 \text{ cm} \quad AD = CD = 8 \text{ cm} \quad \angle ABC = 90^\circ$$

- (a) Find the exact length of AC

Give your answer in the form $p\sqrt{q}$ cm where p is an integer and q is a prime number.

(2)

The point M is the midpoint of AC

- (b) Find the exact length of BM

Give your answer in the form $m\sqrt{n}$ cm where both m and n are prime numbers.

(2)

Given that $BD = 6$ cm,

- (c) find, in degrees to one decimal place, the size of the acute angle between the plane ACD and the plane ABC

(4)

The base ABC of the pyramid is placed on a horizontal plane.

- (d) Find, in cm to 3 significant figures, the vertical height of D above the base.

(2)

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Question 9 continued

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Question 9 continued

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Question 9 continued

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10 (a) Show that $\frac{3}{\sqrt{9-3x}} = \left(1 - \frac{x}{3}\right)^{-\frac{1}{2}}$ (2)

(b) Hence expand $\frac{3}{\sqrt{9-3x}}$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as an exact fraction in its lowest terms. (3)

$$f(x) = \frac{1+2x}{\sqrt{9-3x}}$$

(c) Find the expansion of $3f(x)$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as an exact fraction in its lowest terms. (4)

(d) Hence, using algebraic integration, obtain an approximation to 6 significant figures for

$$\int_{0.1}^{0.2} \frac{1+2x}{\sqrt{9-3x}} dx$$
 (4)

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Question 10 continued

Handwriting practice area with 25 horizontal dotted lines.

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Question 10 continued

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Question 10 continued

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11 The points A and B have coordinates $(-3, -5)$ and $(7, 5)$ respectively.

(a) Find an equation for the line AB (2)

The point C has coordinates $(p, 1)$ where $p < 0$

Given that AC and BC are perpendicular,

(b) prove that $p = -5$ (7)

The point D , where BCD is a straight line, is such that C divides BD in the ratio 4 : 3

(c) Find the coordinates of D (2)

(d) (i) Find the exact length of AC

(ii) Hence, or otherwise, find the area of triangle ABD (4)

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Question 11 continued

Handwriting practice area with 20 horizontal dotted lines.

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