OCR
Oxford Cambridge and RSA

Thursday 7 October 2021 – Afternoon

AS Level Further Mathematics A

Y532/01 Statistics

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 The discrete random variable A has the following probability distribution.

a	1	2	5	10	20
$\mathrm{P}(A = a)$	0.3	0.1	0.1	0.2	0.3

(a) Find the value of $\mathrm{E}(A)$. **[2]**

(b) Determine the value of $\mathrm{Var}(A)$. **[3]**

(c) The variable A represents the value in pence of a coin chosen at random from a pile. Mia picks one coin at random from the pile. She then adds, from a different source, another coin of the same value as the one that she has chosen, and one 50p coin.

(i) Find the mean of the value of the three coins. **[2]**

(ii) Find the variance of the value of the three coins. **[1]**

2 A shopper estimates the cost, £X per item, of each of 12 items in a supermarket. The shopper's estimates are compared with the actual cost, £Y per item, of each item. The results are summarised as follows.

$n = 12$ $\quad\quad \sum x = 399$ $\quad\quad \sum y = 623.88$

$\sum x^2 = 28\,127$ $\quad\quad \sum y^2 = 116\,509.0212$ $\quad\quad \sum xy = 45\,006.01$

Test at the 1% significance level whether the shopper's estimates are positively correlated with the actual cost of the items. **[7]**

3 **(a)** Using the scatter diagram in the Printed Answer Booklet, explain what is meant by **least squares** in the context of a regression line of y on x. **[2]**

(b) A set of bivariate data (t, u) is summarised as follows.

$n = 5$ $\quad\quad \sum t = 35$ $\quad\quad \sum u = 54$

$\sum t^2 = 285$ $\quad\quad \sum u^2 = 758$ $\quad\quad \sum tu = 460$

(i) Calculate the equation of the regression line of u on t. **[3]**

(ii) The variables t and u are now scaled using the following scaling.

$v = 2t, w = u + 4$

Find the equation of the regression line of w on v, giving your equation in the form $w = \mathrm{f}(v)$. **[2]**

4 Two random variables X and Y have the distributions $B(m, p)$ and $B(n, p)$ respectively, where $p > 0$. It is known that

- $E(Y) = 2E(X)$
- $Var(Y) = 1.2E(X)$.

Determine the value of p. **[4]**

5 The discrete random variable X has a geometric distribution. It is given that $Var(X) = 20$.

Determine $P(X \geqslant 7)$. **[6]**

6 A student believes that if you ask people to choose an integer between 1 and 10, not all integers are equally likely to be chosen. The student asks a random sample of 100 people to choose an integer between 1 and 10 inclusive. The observed frequencies O, together with the values of $\frac{(O-E)^2}{E}$ where E is the corresponding expected frequency, are shown in the table.

Integer	1	2	3	4	5	6	7	8	9	10
O	7	8	20	8	7	6	19	7	8	10
$\frac{(O-E)^2}{E}$	0.9	0.4	10.0	0.4	0.9	1.6	8.1	0.9	0.4	0

(a) Show how the value of 8.1 for integer 7 is obtained. **[2]**

(b) Show that there is evidence at the 1% significance level that the student's belief is correct. **[5]**

The student wishes to suggest an alternative model for the probabilities associated with each integer. In this model, two of the integers have the same probability p_1 of being chosen and the other eight integers each have probability p_2 of being chosen.

(c) Suggest which two integers should have probability p_1 and suggest a possible value of p_1. **[2]**

7 The 20 members of a club consist of 10 Town members and 10 Country members.

(a) All 20 members are arranged randomly in a straight line.

Determine the probability that the Town members and the Country members alternate. **[4]**

(b) Ten members of the club are chosen at random.

Show that the probability that the number of Town members chosen is no more than r, where $0 \leqslant r \leqslant 10$, is given by

$$\frac{1}{N}\sum_{i=0}^{r}\left({}^{10}C_i\right)^2$$

where N is an integer to be determined. **[4]**

8 (a) A substance emits particles randomly at a constant average rate of 3.2 per minute. A second substance emits particles randomly, and independently of the first source, at a constant average rate of 2.7 per minute.

Find the probability that the total number of particles emitted by the two sources in a ten-minute period is less than 70. **[3]**

(b) The random variable X represents the number of particles emitted by a substance in a fixed time interval t minutes. It may be assumed that particles are emitted randomly and independently of each other.

In general, the rate at which particles are emitted is proportional to the mass of the substance, but each particle emitted reduces the mass of the substance.

Explain why a Poisson distribution may **not** be a valid model for X if the value of t is very large. **[1]**

(c) The random variable Y has the distribution $\mathrm{Po}(\lambda)$. It is given that

$\mathrm{P}(Y = r) = \mathrm{P}(Y = r + 1)$

$\mathrm{P}(Y = r) = 1.5 \times \mathrm{P}(Y = r - 1)$.

Determine the following, in either order.

- The value of r
- The value of λ **[7]**

END OF QUESTION PAPER