AQA

Please write clearly in block capitals.

Centre number
Candidate number

Surname

Forename(s)

Candidate signature
I declare this is my own work.

AS
FURTHER MATHEMATICS

Paper 1

Monday 15 May 2023 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
TOTAL	

Do not write outside the box

Answer **all** questions in the spaces provided.

1 Which expression below is equivalent to $\tanh x$?

Circle your answer.

[1 mark]

$\sinh x \cosh x$ $\qquad \dfrac{\sinh x}{\cosh x}$ $\qquad \dfrac{\cosh x}{\sinh x}$ $\qquad \sinh x + \cosh x$

2 The two vectors **a** and **b** are such that **a.b** $= 0$

State the angle between the vectors **a** and **b**

Circle your answer.

[1 mark]

0° 45° 90° 180°

Do not write outside the box

3 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 0 & 5 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 & 4 \\ 7 & 1 \end{bmatrix}$$

Calculate **AB**

Circle your answer.

[1 mark]

$\begin{bmatrix} 3 & 5 \\ 7 & 6 \end{bmatrix}$ $\begin{bmatrix} 0 & 20 \\ 21 & 12 \end{bmatrix}$ $\begin{bmatrix} 0 & 4 \\ 0 & 5 \end{bmatrix}$ $\begin{bmatrix} 7 & 13 \\ 35 & 5 \end{bmatrix}$

4 The roots of the equation

$$5x^3 + 2x^2 - 3x + p = 0$$

are α, β and γ

Given that p is a constant, state the value of $\alpha\beta + \beta\gamma + \gamma\alpha$

Circle your answer.

[1 mark]

$-\frac{3}{5}$ $-\frac{2}{5}$ $\frac{2}{5}$ $\frac{3}{5}$

Turn over ▶

Do not write outside the box

5 The function f is defined by

$$f(x) = 3x^2 \qquad 1 \le x \le 5$$

5 (a) Find the mean value of f

[2 marks]

5 (b) The function g is defined by

$$g(x) = f(x) + c \qquad 1 \le x \le 5$$

The mean value of g is 40

Calculate the value of the constant c

[2 marks]

6 (a) Find and simplify the first five terms in the Maclaurin series for e^{2x}

[2 marks]

6 (b) Hence, or otherwise, write down the first five terms in the Maclaurin series for e^{-2x}

[1 mark]

6 (c) Hence, or otherwise, show that the Maclaurin series for $\cosh(2x)$ is

$$a + bx^2 + cx^4 + \ldots$$

where a, b and c are rational numbers to be determined.

[3 marks]

Do not write outside the box

7 (a) Show that, for all integers r,

$$\frac{1}{2r-1} - \frac{1}{2r+1} = \frac{2}{(2r-1)(2r+1)}$$

[1 mark]

7 (b) Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{an}{bn+c}$$

where a, b and c are integers to be determined.

[4 marks]

7 (c) Hence, or otherwise, evaluate

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \ \ldots \ + \frac{1}{99 \times 101}$$

[2 marks]

Do not write outside the box

8 Abdoallah wants to write the complex number $-1 + \mathrm{i}\sqrt{3}$ in the form $r(\cos\theta + \mathrm{i}\sin\theta)$

where $r \geq 0$ and $-\pi < \theta \leq \pi$

Here is his method:

$$r = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2} \qquad \tan\theta = \frac{\sqrt{3}}{-1}$$

$$= \sqrt{1+3} \qquad \Rightarrow \tan\theta = -\sqrt{3}$$

$$= \sqrt{4} \qquad \Rightarrow \theta = \tan^{-1}\left(-\sqrt{3}\right)$$

$$= 2 \qquad \Rightarrow \theta = -\frac{\pi}{3}$$

$$-1 + \mathrm{i}\sqrt{3} = 2\left(\cos\left(-\frac{\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{\pi}{3}\right)\right)$$

There is an error in Abdoallah's method.

8 (a) Show that Abdoallah's answer is wrong by writing

$$2\left(\cos\left(-\frac{\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{\pi}{3}\right)\right)$$

in the form $x + \mathrm{i}y$

Simplify your answer.

[1 mark]

Do not write outside the box

8 (b) Explain the error in Abdoallah's method.

[1 mark]

8 (c) Express $-1 + i\sqrt{3}$ in the form $r(\cos\theta + i\sin\theta)$

[1 mark]

8 (d) Write down the complex conjugate of $-1 + i\sqrt{3}$

[1 mark]

Do not write outside the box

9 The matrix **M** represents the transformation T and is given by

$$\mathbf{M} = \begin{bmatrix} 3p+1 & 12 \\ p+2 & p^2-3 \end{bmatrix}$$

9 (a) In the case when $p = 0$ show that the image of the point (4, 5) under T is the point (64, –7)

[2 marks]

9 (b) In the case when $p = -2$ find the gradient of the line of **invariant points** under T

[3 marks]

Do not write outside the box

9 (c) Show that $p = 3$ is the only real value of p for which **M** is singular.

[6 marks]

Turn over ▶

Do not write outside the box

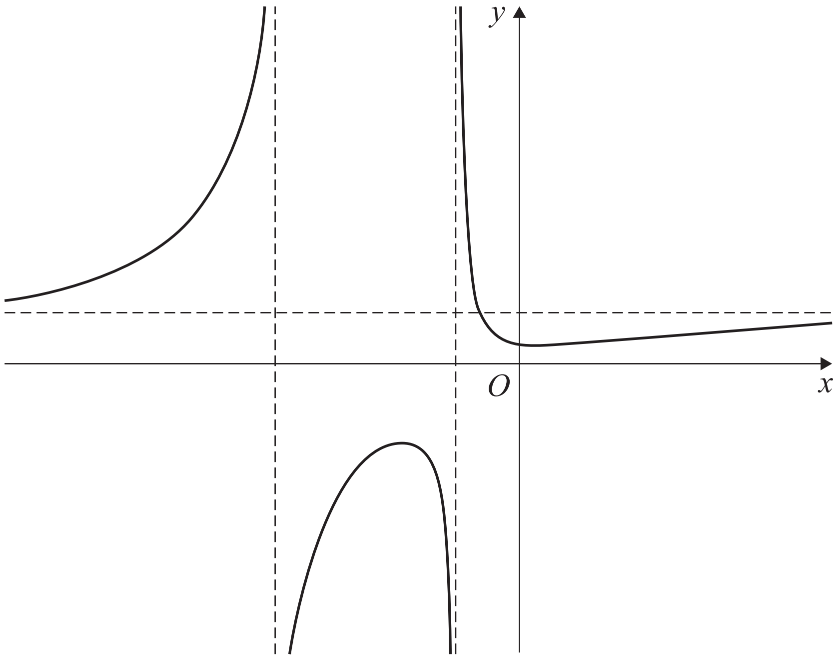
10 The curve C has equation

$$y = \frac{3x^2 + mx + p}{x^2 + px + m}$$

where m and p are integers.

The vertical asymptotes of C are $x = -4$ and $x = -1$

The curve C is shown in the diagram below.



10 (a) Write down the equation of the horizontal asymptote of C

[1 mark]

---

10 (b) Find the value of m and the value of p

[2 marks]

---

10 (c) Hence, or otherwise, write down the coordinates of the y-intercept of C

[1 mark]

10 (d) Without using calculus, show that the line $y = -1$ does not intersect C

[5 marks]

11 A point has Cartesian coordinates (x, y) and polar coordinates (r, θ) where $r \geq 0$ and $-\pi < \theta \leq \pi$

11 (a) Express r in terms of x and y

[1 mark]

11 (b) Express x in terms of r and θ

[1 mark]

11 (c) The curve C_1 has the polar equation

$$r(2 + \cos\theta) = 1 \qquad -\pi < \theta \leq \pi$$

11 (c) (i) Show that the Cartesian equation of C_1 can be written as

$$ay^2 = (1 + bx)(1 + x)$$

where a and b are integers to be determined.

[4 marks]

Do not write outside the box

11 (c) (ii) The curve C_2 has the Cartesian equation

$$ax^2 = (1 + by)(1 + y)$$

where a and b take the same values as in part **(c)(i)**.

Describe fully a single transformation that maps the curve C_1 onto the curve C_2

[2 marks]

Do not write outside the box

12 (a) Show that $(1 + \mathrm{i})^4 = -4$

[3 marks]

12 (b) The function f is defined by

$$\mathrm{f}(z) = z^4 + 3z^2 - 6z + 10 \qquad z \in \mathbb{C}$$

12 (b) (i) Show that $(1 + \mathrm{i})$ is a root of $\mathrm{f}(z) = 0$

[2 marks]

12 (b) (ii) Hence write down another root of $\mathrm{f}(z) = 0$

[1 mark]

Do not write outside the box

12 (b) (iii) One of the linear factors of $\mathrm{f}(z)$ is

$$\big(z - (1 + \mathrm{i})\big)$$

Write down another linear factor and hence, or otherwise, find a quadratic factor of $\mathrm{f}(z)$ with real coefficients.

[3 marks]

12 (b) (iv) Find another quadratic factor of $\mathrm{f}(z)$ with real coefficients.

[2 marks]

12 (b) (v) Hence explain why the graph of $y = \mathrm{f}(x)$ does not intersect the x-axis.

[2 marks]

Turn over ▶

Do not write outside the box

13 (a) Prove by induction that, for all integers $n \geq 1$,

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

[4 marks]

Do not write outside the box

13 (b) Hence, or otherwise, write down a factorised expression for the sum of the first $2n$ squares

$$1^2 + 2^2 + 3^2 + \ldots + (2n)^2$$

[1 mark]

13 (c) Use the formula in part **(a)** to write down a factorised expression for the sum of the first n even squares

$$2^2 + 4^2 + 6^2 + \ldots + (2n)^2$$

[2 marks]

13 (d) Hence, or otherwise, show that the sum of the first n odd squares is

$$an(bn - 1)(bn + 1)$$

where a and b are rational numbers to be determined.

[3 marks]

Turn over ▶

14 The inequality

$$(x^2 - 5x - 24)(x^2 + 7x + a) < 0$$

has the solution set

$$\{x : -9 < x < -3\} \cup \{x : 2 < x < b\}$$

Find the values of integers a and b

[4 marks]

END OF QUESTIONS

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	Additional page, if required. Write the question numbers in the left-hand margin.

Do not write outside the box

Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**