



Oxford Cambridge and RSA

Monday 3 June 2019 – Morning

A Level Further Mathematics A

Y540/01 Pure Core 1

Time allowed: 1 hour 30 minutes



You must have:

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 In this question you must show detailed reasoning.

The quadratic equation $x^2 - 2x + 5 = 0$ has roots α and β .

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [1]

(b) Hence find a quadratic equation with roots $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$. [3]

2 Indicate by shading on an Argand diagram the region

$\{z : |z| \leq |z - 4|\} \cap \{z : |z - 3 - 2i| \leq 2\}$. [3]

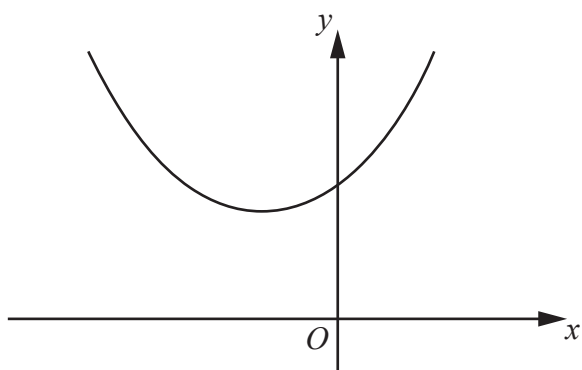
3 In this question you must show detailed reasoning.

You are given that $x = 2 + 5i$ is a root of the equation $x^3 - 2x^2 + 21x + 58 = 0$.

Solve the equation. [4]

4 Using the formulae for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$, show that $\sum_{r=1}^{10} r(3r - 2) = 1045$. [3]

- 5 The diagram shows part of the curve $y = 5 \cosh x + 3 \sinh x$.



- (a) Solve the equation $5 \cosh x + 3 \sinh x = 4$ giving your solution in exact form. [4]
- (b) **In this question you must show detailed reasoning.**

Find $\int_{-1}^1 (5 \cosh x + 3 \sinh x) dx$ giving your answer in the form $ae + \frac{b}{e}$ where a and b are integers to be determined. [3]

- 6 You are given that $y = \tan^{-1} \sqrt{2x}$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Show that $\int_{\frac{1}{6}}^{\frac{1}{2}} \frac{\sqrt{x}}{(x+2x^2)} dx = k\pi$ where k is a number to be determined in exact form. [4]

- 7 The function $\operatorname{sech} x$ is defined by $\operatorname{sech} x = \frac{1}{\cosh x}$.

(a) Show that $\operatorname{sech} x = \frac{2e^x}{e^{2x} + 1}$. [2]

(b) Using a suitable substitution, find $\int \operatorname{sech} x dx$. [4]

- 8 The equation of a plane is $4x + 2y + z = 7$.
The point A has coordinates $(9, 6, 1)$ and the point B is the reflection of A in the plane.

Find the coordinates of the point B .

[6]

9 **In this question you must show detailed reasoning.**

You are given the complex number $\omega = \cos\frac{2}{5}\pi + i\sin\frac{2}{5}\pi$ and the equation $z^5 = 1$.

(a) Show that ω is a root of the equation. [2]

(b) Write down the other four roots of the equation. [1]

(c) Show that $\omega + \omega^2 + \omega^3 + \omega^4 = -1$. [2]

(d) Hence show that $\left(\omega + \frac{1}{\omega}\right)^2 + \left(\omega + \frac{1}{\omega}\right) - 1 = 0$. [3]

(e) Hence determine the value of $\cos\frac{2}{5}\pi$ in the form $a + b\sqrt{c}$ where a , b and c are rational numbers to be found. [4]

10 You are given the matrix \mathbf{A} where $\mathbf{A} = \begin{pmatrix} a & 2 & 0 \\ 0 & a & 2 \\ 4 & 5 & 1 \end{pmatrix}$.

(a) Find, in terms of a , the determinant of \mathbf{A} , simplifying your answer. [2]

(b) Hence find the values of a for which \mathbf{A} is singular. [2]

You are given the following equations which are to be solved simultaneously.

$$ax + 2y = 6$$

$$ay + 2z = 8$$

$$4x + 5y + z = 16$$

(c) For each of the values of a found in part (b) determine whether the equations have

- a unique solution, which should be found, or
- an infinite set of solutions or
- no solution.

[7]

- 11 A particle is suspended in a resistive medium from one end of a light spring. The other end of the spring is attached to a point which is made to oscillate in a vertical line.

The displacement of the particle may be modelled by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 10 \sin t$$

where x is the displacement of the particle below the equilibrium position at time t .

When $t = 0$ the particle is stationary and its displacement is 2.

- (a) Find the particular solution of the differential equation. [11]
- (b) Write down an approximate equation for the displacement when t is large. [2]

END OF QUESTION PAPER

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