Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/4D**

Further Mathematics

Advanced

PAPER 4D: Decision Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator,
Decision Mathematics Answer Book (enclosed)

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. Four workers, A, B, C and D, are to be assigned to four tasks, 1, 2, 3 and 4. Each task must be assigned to just one worker and each worker must do only one task.

The cost of assigning each worker to each task is shown in the table below.

The total cost is to be minimised.

	1	2	3	4
A	32	45	34	48
B	37	39	50	46
C	46	44	40	42
D	43	45	48	52

(a) Reducing rows first, use the Hungarian algorithm to obtain an allocation that minimises the total cost. You must make your method clear and show the table after each stage.

(5)

(b) State the minimum total cost.

(1)

(Total for Question 1 is 6 marks)

2. The general solution of the second order recurrence relation

$$u_{n+2} + k_1 u_{n+1} + k_2 u_n = 0 \qquad n \geqslant 0$$

is given by

$$u_n = (A + Bn)(-3)^n$$

where A and B are arbitrary non-zero constants.

(a) Find the value of k_1 and the value of k_2

(2)

Given that $u_0 = u_1 = 1$

(b) find the value of A and the value of B.

(2)

(Total for Question 2 is 4 marks)

3. The table below shows the transport options, usual travel times, possible delay times and corresponding probabilities of delay for a journey. All times are in minutes.

Transport option	Usual travel time	Possible delay time	Probability of delay
Car	52	10	0.10
		25	0.02
Train	45	15	0.05
		25	0.03
Coach	55	5	0.05
		15	0.01

(a) Draw a decision tree to model the transport options and the possible outcomes.

(5)

(b) State the minimum expected travel time and the corresponding transport option indicated by the decision tree.

(2)

(Total for Question 3 is 7 marks)

4.

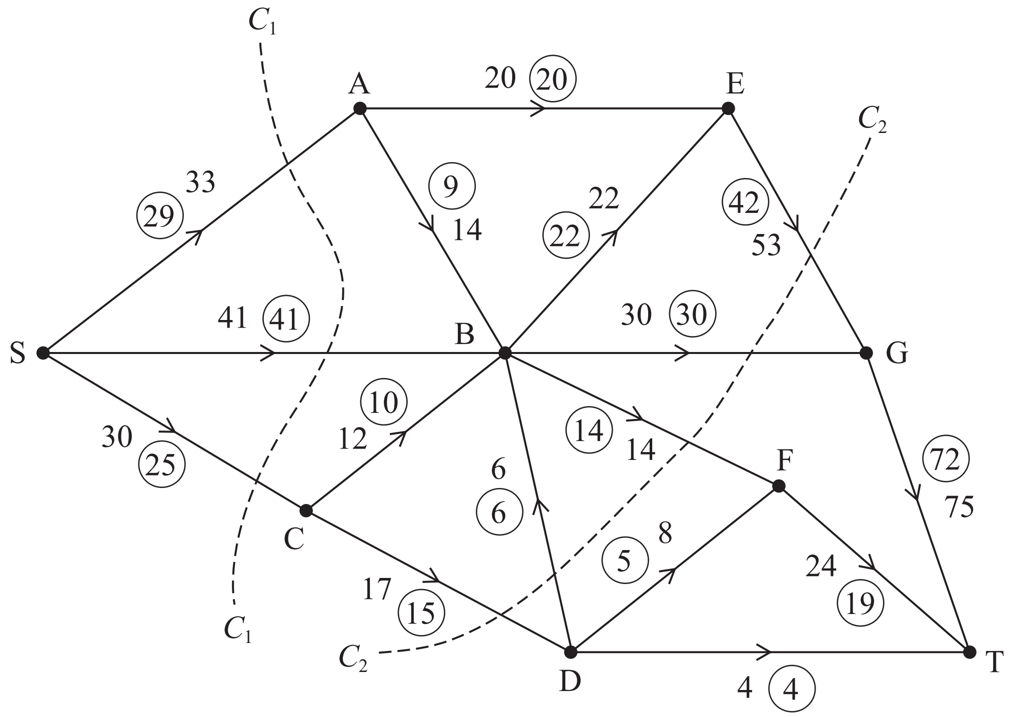

Figure 1

Figure 1 shows a capacitated, directed network of pipes. The uncircled number on each arc represents the capacity of the corresponding pipe. The numbers in circles represent an initial flow.

(a) List the saturated arcs.

(1)

(b) State the value of the initial flow.

(1)

(c) Explain why arc FT cannot be full to capacity.

(1)

(d) State the capacity of cut C_1 and the capacity of cut C_2

(2)

(e) By inspection find one flow-augmenting route to increase the flow by three units. You must state your route.

(1)

(f) Prove that, once the flow-augmenting route found in part (e) has been applied, the flow is maximal.

(3)

(Total for Question 4 is 9 marks)

5. A standard transportation problem is described in the linear programming formulation below.

Let x_{ij} be the number of units transported from i to j

where $i \in \{\text{A, B, C, D}\}$

$j \in \{\text{R, S, T}\}$ and $x_{ij} \geqslant 0$

Minimise $P = 23x_{\text{AR}} + 17x_{\text{AS}} + 24x_{\text{AT}} + 15x_{\text{BR}} + 29x_{\text{BS}} + 32x_{\text{BT}}$

$+ 25x_{\text{CR}} + 25x_{\text{CS}} + 27x_{\text{CT}} + 19x_{\text{DR}} + 20x_{\text{DS}} + 25x_{\text{DT}}$

subject to

$$\sum x_{\text{A}j} \leqslant 34$$

$$\sum x_{\text{B}j} \leqslant 27$$

$$\sum x_{\text{C}j} \leqslant 41$$

$$\sum x_{\text{D}j} \leqslant 18$$

$$\sum x_{i\text{R}} \geqslant 44$$

$$\sum x_{i\text{S}} \geqslant 37$$

$$\sum x_{i\text{T}} \geqslant k$$

Given that the problem is balanced,

(a) state the value of k.

(1)

(b) Explain precisely what the constraint $\sum x_{i\text{R}} \geqslant 44$ means in the transportation problem.

(2)

(c) Use the north-west corner method to obtain the cost of an initial solution to this transportation problem.

(2)

(d) Perform one iteration of the stepping-stone method to obtain an improved solution. You must make your method clear by showing the route and the

- shadow costs
- improvement indices
- entering cell and exiting cell.

(4)

(Total for Question 5 is 9 marks)

6. Bernie makes garden sheds. He can build up to four sheds each month.

If he builds more than two sheds in any one month, he must hire an additional worker at a cost of £250 for that month.

In any month in which sheds are made, the overhead costs are £35 for each shed made that month.

A maximum of three sheds can be held in storage at the end of any one month, at a cost of £80 per shed per month.

Sheds must be delivered at the end of the month.

The order schedule for sheds is

Month	January	February	March	April	May
Number ordered	1	3	3	5	2

There are no sheds in storage at the beginning of January and Bernie plans to have no sheds left in storage after the May delivery.

Use dynamic programming to determine the production schedule that minimises the costs given above. Complete the working in the table provided in the answer book and state the minimum cost.

(14)

(Total for Question 6 is 14 marks)

7.

		Player B			
		Option W	Option X	Option Y	Option Z
Player A	Option Q	4	3	–1	–2
	Option R	–3	5	–4	k
	Option S	–1	6	3	–3

A two person zero-sum game is represented by the pay-off matrix for player A shown above. It is given that k is an integer.

(a) Show that Q is the play-safe option for player A regardless of the value of k.

(2)

Given that Z is the play-safe option for player B,

(b) determine the range of possible values of k. You must make your working clear.

(2)

(c) Explain why player B should never play option X. You must make your reasoning clear.

(2)

Player A intends to make a random choice between options Q, R and S, choosing option Q with probability p_1, option R with probability p_2 and option S with probability p_3

Player A wants to find the optimal values of p_1, p_2 and p_3 using the Simplex algorithm.

Given that $k > -4$, player A formulates the following objective function for the corresponding linear program.

Maximise $P = V$, where V = the value of the original game + 4

(d) (i) Formulate the constraints of the linear programming problem for player A. You should write the constraints as equations.

(ii) Write down an initial Simplex tableau, making your variables clear.

(7)

The Simplex algorithm is used to solve the linear programming problem. It is given that in the final Simplex tableau the optimal value of $p_1 = \frac{7}{37}$, the optimal value of $p_2 = \frac{17}{37}$ and all the slack variables are zero.

(e) Determine the value of k, making your method clear.

(4)

(Total for Question 7 is 17 marks)

8. The owner of a new company models the number of customers that the company will have at the end of each month. The owner assumes that

- a constant proportion, p (where $0 < p < 1$), of the previous month's customers will be retained for the next month
- a constant number of new customers, k, will be added each month.

Let u_n (where $n \geqslant 1$) represent the number of customers that the company will have at the end of n months.

The company has 5000 customers at the end of the first month.

(a) By setting up a first order recurrence relation for u_{n+1} in terms of u_n, determine an expression for u_n in terms of n, p and k.

(6)

The owner believes that 95% of the previous month's customers will be retained each month and that there will be 10 000 new customers each month.

According to the model, the company will first have at least 135 000 customers by the end of the mth month.

(b) Using logarithms, determine the value of m.

(3)

(Total for Question 8 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS