AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS MATHEMATICS

Paper 1

Thursday 18 May 2023 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 At a point P on a curve, the gradient of the tangent to the curve is 10

State the gradient of the normal to the curve at P

Circle your answer.

[1 mark]

-10 -0.1 0.1 10

2 Identify the expression below which is equivalent to $\left(\frac{2x}{5}\right)^{-3}$

Circle your answer.

[1 mark]

$\frac{8x^3}{125}$ $\frac{125x^3}{8}$ $\frac{125}{8x^3}$ $\frac{8}{125x^3}$

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3 The coefficient of x^2 in the binomial expansion of $(1 + ax)^6$ is $\frac{20}{3}$

Find the two possible values of a

[3 marks]

Turn over for the next question

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4 It is given that $5\cos^2\theta - 4\sin^2\theta = 0$

4 (a) Find the possible values of $\tan\theta$, giving your answers in exact form.

[3 marks]

4 (b) Hence, or otherwise, solve the equation

$$5\cos^2\theta - 4\sin^2\theta = 0$$

giving all solutions of θ to the nearest 0.1° in the interval $0° \leq \theta \leq 360°$

[2 marks]

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5 (a) Given that $y = x\sqrt{x}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

[2 marks]

5 (b) The line, *L*, has equation $6x - 2y + 5 = 0$

L is a tangent to the curve with equation $y = x\sqrt{x} + k$

Find the value of k

[5 marks]

6 (a) The curve C_1 has equation $y = 2x^2 - 20x + 42$

Express the equation of C_1 in the form

$$y = a(x - b)^2 + c$$

where a, b and c are integers.

[3 marks]

6 (b) Write down the coordinates of the minimum point of C_1

[1 mark]

6 (c) The curve C_1 is mapped onto the curve C_2 by a stretch in the y-direction.

The minimum point of C_2 is at $(5, -4)$

Find the equation of C_2

[2 marks]

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7 Points *P* and *Q* lie on the curve with equation $y = x^4$

The x-coordinate of *P* is x
The x-coordinate of *Q* is $x + h$

7 (a) Expand $(x + h)^4$

[2 marks]

7 (b) Hence, find an expression, in terms of x and h, for the gradient of the line *PQ*

[1 mark]

7 (c) Explain how to use the answer from part **(b)** to obtain the gradient function of $y = x^4$

[2 marks]

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8 (a) Show that

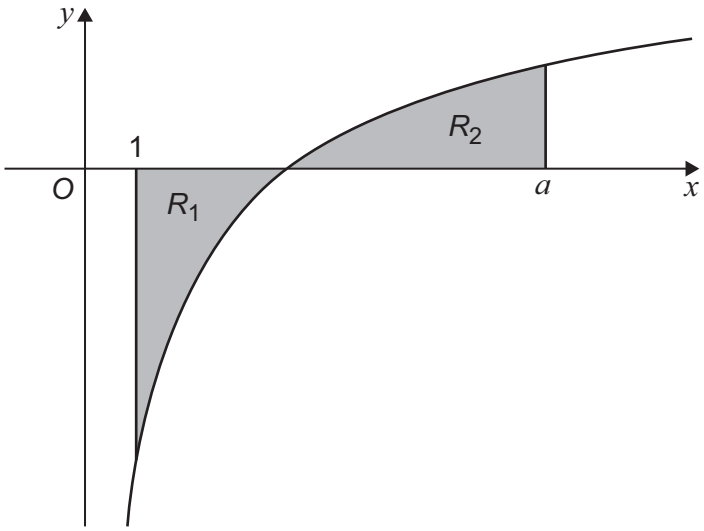
$$\int_1^a \left(6 - \frac{12}{\sqrt{x}}\right) \mathrm{d}x = 6a - 24\sqrt{a} + 18$$

[3 marks]

8 (b) The curve $y = 6 - \frac{12}{\sqrt{x}}$, the line $x = 1$ and the line $x = a$ are shown in the diagram below.

The shaded region R_1 is bounded by the curve, the line $x = 1$ and the x-axis.

The shaded region R_2 is bounded by the curve, the line $x = a$ and the x-axis.



It is given that the areas of R_1 and R_2 are equal.

Find the value of a

Fully justify your answer.

[4 marks]

9 A continuous curve has equation $y = \mathrm{f}(x)$

The curve passes through the points $A(2, 1)$, $B(4, 5)$ and $C(6, 1)$

It is given that $\mathrm{f}'(4) = 0$

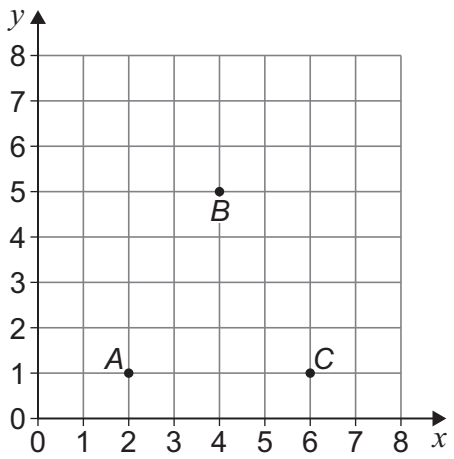
Jasmin made two statements about the nature of the curve $y = \mathrm{f}(x)$ at the point B:

Statement 1: There is a turning point at B

Statement 2: There is a maximum point at B

9 (a) Draw a sketch of the curve $y = \mathrm{f}(x)$ such that Statement 1 is correct and Statement 2 is correct.

[1 mark]



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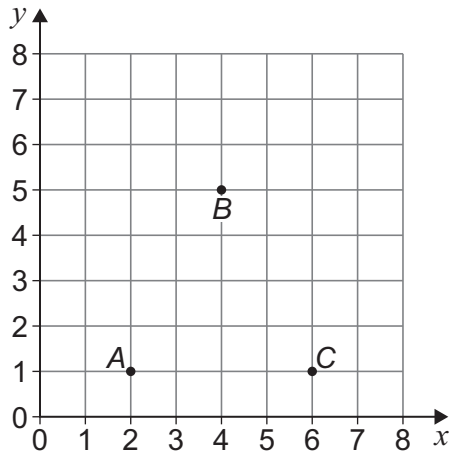
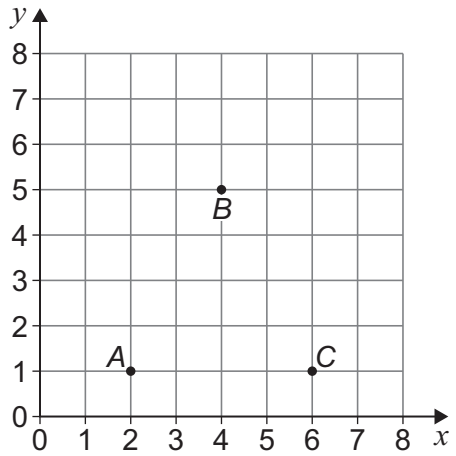
9 (b) Draw a sketch of the curve $y = f(x)$ such that Statement 1 is correct and Statement 2 is **not** correct.

[1 mark]



9 (c) Draw a sketch of the curve $y = f(x)$ such that Statement 1 is **not** correct and Statement 2 is **not** correct.

[1 mark]



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10 Charlie buys a car for £18 000 on 1 January 2016.

The value of the car decreases exponentially.

The car has a value of £12 000 on 1 January 2018.

10 (a) Charlie says:

- because the car has lost £6000 after two years, after another two years it will be worth £6000.

Charlie's friend Kaya says:

- because the car has lost one third of its value after two years, after another two years it will be worth £8000.

Explain whose statement is correct, justifying the value they have stated.

[2 marks]

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10 (b) The value of Charlie's car, £V, t years after 1 January 2016 may be modelled by the equation

$$V = A\mathrm{e}^{-kt}$$

where A and k are positive constants.

Find the value of t when the car has a value of £10 000, giving your answer to two significant figures.

[5 marks]

10 (c) Give a reason why the model, in this context, will not be suitable to calculate the value of the car when $t = 30$

[1 mark]

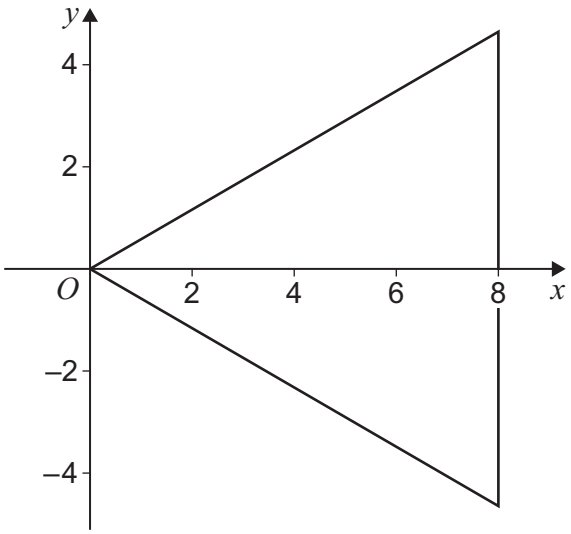
11 (a) A circle has equation

$$x^2 + y^2 - 10x - 6 = 0$$

Find the centre and the radius of the circle.

[2 marks]

11 (b) An equilateral triangle has one vertex at the origin, and one side along the line $x = 8$, as shown in the diagram below.



11 (b) (i) Show that the vertex at the origin lies inside the circle $x^2 + y^2 - 10x - 6 = 0$

[1 mark]

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11 (b) (ii) Prove that the triangle lies completely within the circle $x^2 + y^2 - 10x - 6 = 0$

[4 marks]

END OF SECTION A

TURN OVER FOR SECTION B

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Section B

Answer **all** questions in the spaces provided.

12 A particle, initially at rest, starts to move forward in a straight line with constant acceleration, $a\,\text{m s}^{-2}$

After 6 seconds the particle has a velocity of $3\,\text{m s}^{-1}$

Find the value of a

Circle your answer.

[1 mark]

-2 -0.5 0.5 2

13 A resultant force of $\begin{bmatrix} -2 \\ 6 \end{bmatrix}$ N acts on a particle.

The acceleration of the particle is $\begin{bmatrix} -6 \\ y \end{bmatrix}\,\text{m s}^{-2}$

Find the value of y

Circle your answer.

[1 mark]

2 3 10 18

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14 A ball, initially at rest, is dropped from a vertical height of h metres above the Earth's surface.

After 4 seconds the ball's height above the Earth's surface is $0.2h$ metres.

14 (a) Assuming air resistance can be ignored, show that

$$h = 10g$$

[3 marks]

14 (b) Assuming air resistance cannot be ignored, explain the effect that this would have on the value of h in part **(a)**.

[1 mark]

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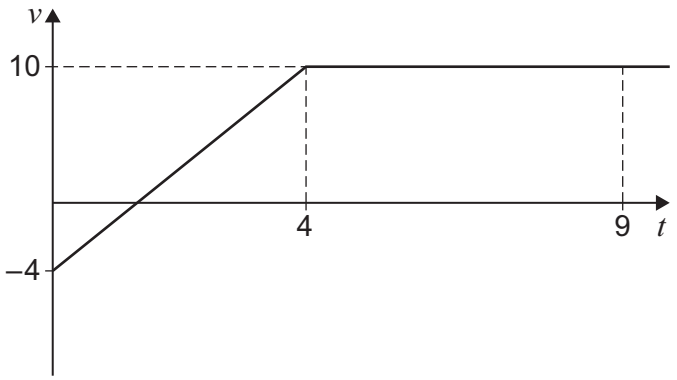
15 A particle is moving in a straight line such that its velocity, $v\,\text{m s}^{-1}$, changes with respect to time, t seconds, as shown in the graph below.



15 (a) Show that the acceleration of the particle over the first 4 seconds is $3.5\,\text{m s}^{-2}$

[1 mark]

15 (b) The particle is initially at a fixed point P

Show that the displacement of the particle from P, when $t = 9$, is 62 metres.

[3 marks]

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16 A toy remote control speed boat is launched from one edge of a small pond and moves in a straight line across the pond's surface.

The boat's velocity, $v\,\text{m}\,\text{s}^{-1}$, is modelled in terms of time, t seconds after the boat is launched, by the expression

$$v = 0.9 + 0.16t - 0.06t^2$$

16 (a) Find the acceleration of the boat when $t = 2$

[3 marks]

16 (b) Find the displacement of the boat, from the point where it was launched, when $t = 2$

[4 marks]

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17 A particle, P, is initially at rest on a smooth horizontal surface.

A resultant force of $\begin{bmatrix} 12 \\ 9 \end{bmatrix}$ N is then applied to P, so that it moves in a straight line.

17 (a) Find the magnitude of the resultant force.

[1 mark]

17 (b) Two fixed points A and B have position vectors

$$\overrightarrow{OA} = \begin{bmatrix} 3 \\ 7 \end{bmatrix} \text{ metres} \quad \text{and} \quad \overrightarrow{OB} = \begin{bmatrix} k \\ k-1 \end{bmatrix} \text{ metres}$$

with respect to a fixed origin, O

P moves in a straight line parallel to $\overrightarrow{AB}$

17 (b) (i) Find $\overrightarrow{AB}$ in terms of k

[1 mark]

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17 (b) (ii) Find the value of k

[2 marks]

Turn over for the next question

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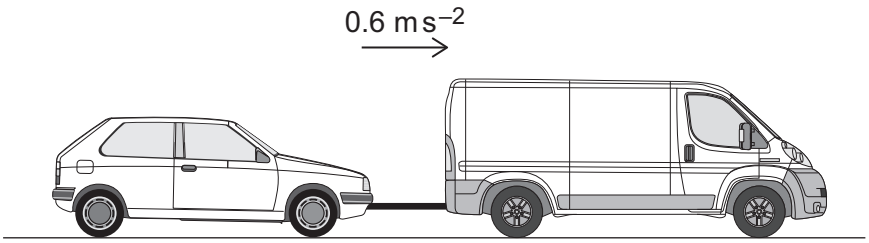
18 A rescue van is towing a broken-down car by using a tow bar.

The van and the car are moving with a constant acceleration of $0.6\,\text{m}\,\text{s}^{-2}$ along a straight horizontal road as shown in the diagram below.



The van has a total mass of 2780 kg

The car has a total mass of 1620 kg

The van experiences a driving force of D newtons.

The van experiences a total resistance force of R newtons.

The car experiences a total resistance force of $0.6R$ newtons.

18 (a) The tension in the tow bar, T newtons, may be modelled by

$$T = kD - 18$$

where k is a constant.

Find k

[5 marks]

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18 (b) State one assumption that must be made in answering part **(a)**.

[1 mark]

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END OF QUESTIONS

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ANSWER IN THE SPACES PROVIDED

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