

GCE

Further Mathematics A

Y535/01: Additional Pure Mathematics

Advanced Subsidiary GCE

2021 Mark Scheme (DRAFT)

This is a DRAFT mark scheme. It has not been used for marking as this paper did not receive any entries in the series it was scheduled for. It is therefore possible that not all valid approaches to a question may be captured in this version. You should give credit to such responses when marking learner's work.

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

© OCR 2021

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

- a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.
 - When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
- NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads “2 s.f”.
- Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
- Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.
- g Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
 - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
 - If a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
- If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” or “Determine”. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)	(i)	$\mathbf{a} \times \mathbf{b} = (3\mathbf{i}) \times (4\mathbf{j}) = 12(\mathbf{i} \times \mathbf{j}) = 12\mathbf{k} = 12\mathbf{c}$	B1 [1]	1.1	
		(ii)	The VP of \mathbf{a} and \mathbf{b} must be perpendicular to both; i.e. in the direction of \mathbf{c}	B1 [1]	2.4	Allow “since \mathbf{a} , \mathbf{b} , \mathbf{c} form a right-handed system” or “since \mathbf{a} , \mathbf{b} , \mathbf{c} form a set of mutually perpendicular vectors”
	(b)	Any two of $\mathbf{b} - \mathbf{a} = -3\mathbf{i} + 4\mathbf{j}$, $\mathbf{c} - \mathbf{a} = -3\mathbf{i} + \mathbf{k}$, $\mathbf{c} - \mathbf{b} = -4\mathbf{j} + \mathbf{k}$ Area $\Delta ABC = \frac{1}{2} \left \overrightarrow{AB} \times \overrightarrow{AC} \right = \frac{1}{2} \left \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix} \right = \frac{13}{2}$	B1 M1 A1 [3]	1.1 1.1 1.1	(\pm) Use of area formula with relevant vectors <i>and</i> attempt at a vector product (condone missing $\frac{1}{2}$) Correct answer	
2	(a)	(i)	E.g. when $y = 1$, $z = x^2 + 4x$	B1 [1]	1.2	$z = x^2 + 4ax$ for any a , $0 < a \leq 2$ Only one chosen case is required. If the general case is offered, the suitable range of values of the chosen parameter must be noted (though condone the inclusion of “ $a = 0$ ”).
		(ii)	(Part of) a \cup -shaped parabola in the x - z plane Thro’ (0,0), with Min \approx at $(-2a, -4a^2)$	M1 A1 [2]	1.1 1.1	(BC) FT Details correct; must exist only for $x: -6 \leq x \leq 6$
	(b)	E.g. when $z = 1$, $1 = x^2 + 4xy$	B1 [1]	1.2	$b = x^2 + 4xy$ for any b , $-16 \leq b \leq 84$, $b \neq 0$ Only one chosen case is required. If the general case is offered, it must only be noted that “ $b \neq 0$ ”.	

Question		Answer	Marks	AO	Guidance	
3	(a)	$F_{3k+3} = F_{3k+2} + F_{3k+1} = (F_{3k+1} + F_{3k}) + F_{3k+1}$ $= 2F_{3k+1} + F_{3k}$	M1 A1 [2]	2.1 1.1	Use of Fibonacci r.r. <i>twice</i> AG	
	(b)	Since $F_3 = F_{3 \times 1} = 2$ is even, the result is true for $n = 1$ Assuming that F_{3k} is even for some $k (\geq 1)$ $F_{3k+3} = \text{even} + \text{even} = \text{even}$ also Since result is true for $n = 1$ and true for $n + 1$ whenever true for n , proof follows by induction	B1 M1 A1 B1 [4]	1.1 2.1 2.2a 2.4	It must be made clear, somewhere, what “assume the result is true for $n = k$ ” means; and must include identification that $F_{3k+3} = F_{3(k+1)}$ is the case for $n = k + 1$ at some stage Use of (a) ’s result, clearly made Carefully explained induction conclusion	
4	(a)	(i)	$1071 \div 67 = 15 \frac{66}{67}$ so $q = 15$ and $r = 66$	B1 [1]	1.1	BC
		(ii)	$1071 \equiv 66 \pmod{67}$	B1 [1]	1.1	FT (a) (i) ’s answer
	(b)	Suppose $\text{hcf}(715, 1071) = h$, so $h \mid 715$ and $h \mid 1071$ $\Rightarrow h \mid 715a + 1071b$ In this case, $a = 358, b = -239$ give $h \mid 1$ $\Rightarrow h = 1$ and 715, 1071 are co-prime	M1 M1 A1 B1 [4]	3.1a 2.1 1.1 2.2a	(for all integers a, b) Must show understanding of conclusion relative to question’s demand. (Condone correct conclusion of primality from $h = \pm 1$.)	

Question	Answer	Marks	AO	Guidance
5 (a)	$\frac{\partial z}{\partial x} = 0.9 - \frac{0.096y}{x^2} - 2xy^2 \quad \frac{\partial z}{\partial y} = \frac{0.096}{x} - 2x^2y$ <p>Setting both $\frac{\partial z}{\partial x} = 0$ and $\frac{\partial z}{\partial y} = 0$ and expressing (say) y in terms of x from the second of these</p> <p>In $\frac{\partial z}{\partial x} = 0$, $0.9 - \frac{0.096}{x^2} \left(\frac{0.048}{x^3} \right) - 2x \left(\frac{0.048}{x^3} \right)^2 = 0$</p> <p>$\Rightarrow x^5 = 0.01024 \Rightarrow x = 0.4$ $\Rightarrow y = 0.75$ and Cost = £4500</p>	<p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[7]</p>	<p>1.1 1.1</p> <p>1.1</p> <p>3.1a</p> <p>1.1</p> <p>1.1</p> <p>3.2a</p>	<p>Attempt at both first p.d.s; at least one correct</p> <p>Must include an attempt to do something with at least one of the two equations: $\frac{\partial z}{\partial y} = 0 \Rightarrow y = \frac{0.048}{x^3}$*</p> <p>Substituting into the other equation and solving attempt</p> <p>FT from 1st variable found in clearly s.o.i. relationship (e.g.*) CAO From correct $z = 0.45$</p>
(b)	$\frac{\partial^2 z}{\partial x^2} = \frac{0.192y}{x^3} - 2y^2 \quad \text{and} \quad \frac{\partial^2 z}{\partial y^2} = -2x^2$ <p style="text-align: center;">$= 1.125 \qquad \qquad \qquad = -0.32$</p> <p>Change in y will decrease z since 2nd-p.d. < 0</p> <hr/> <p>Alternative method</p> <p>When with $y = 0.75$, $x = 0.39, 0.4, 0.41$ $z = 0.450\ 059, 0.45, 0.450\ 054$ and when $x = 0.4$, $y = 0.74, 0.75, 0.76$ $z = 0.449\ 984, 0.45, 0.449\ 984$ Change in y will decrease z</p>	<p>M1</p> <p>A1 A1</p> <p>B1</p> <hr/> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p>	<p>3.3</p> <p>3.4 1.1</p> <p>3.5a</p> <hr/> <p></p> <p></p> <p></p> <p></p>	<p>Attempt at the two second partial differentials</p> <p>Correct value of z_{xx} & value or sign statement for a suitable z_{yy} Correct answer for correct reason (FT +ve and -ve if values are not offered) NB $z_{xx} > 0 \Rightarrow$ the SV is a min. in the x-direction, while $z_{yy} < 0 \Rightarrow$ the SV is a max. in the y-direction</p> <p>Adjusting values of x with y fixed; then y with x fixed Correct 1st set of results</p> <p>Correct 2nd set of results</p> <p>Correct answer for correct reason When with $y = 0.75$, $x = 0.35, 0.4, 0.45$ $z = 0.451\ 808, 0.45, 0.451\ 094$ and when $x = 0.4$, $y = 0.7, 0.75, 0.8$ $z = 0.449\ 6, 0.45, 0.449\ 6$</p>

Question	Answer	Marks	AO	Guidance
		[4]		

Question	Answer	Marks	AO	Guidance																										
6	(a) (i)	<table border="1"> <tr> <td></td> <td>$\sqrt{3} + i$</td> <td>$-\sqrt{3} - i$</td> <td>$1 - i\sqrt{3}$</td> <td>$-1 + i\sqrt{3}$</td> </tr> <tr> <td>$\sqrt{3} + i$</td> <td>$-1 + i\sqrt{3}$</td> <td>$1 - i\sqrt{3}$</td> <td>$\sqrt{3} + i$</td> <td>$-\sqrt{3} - i$</td> </tr> <tr> <td>$-\sqrt{3} - i$</td> <td>$1 - i\sqrt{3}$</td> <td>$-1 + i\sqrt{3}$</td> <td>$-\sqrt{3} - i$</td> <td>$\sqrt{3} + i$</td> </tr> <tr> <td>$1 - i\sqrt{3}$</td> <td>$\sqrt{3} + i$</td> <td>$-\sqrt{3} - i$</td> <td>$1 - i\sqrt{3}$</td> <td>$-1 + i\sqrt{3}$</td> </tr> <tr> <td>$-1 + i\sqrt{3}$</td> <td>$-\sqrt{3} - i$</td> <td>$\sqrt{3} + i$</td> <td>$-1 + i\sqrt{3}$</td> <td>$1 - i\sqrt{3}$</td> </tr> </table>		$\sqrt{3} + i$	$-\sqrt{3} - i$	$1 - i\sqrt{3}$	$-1 + i\sqrt{3}$	$\sqrt{3} + i$	$-1 + i\sqrt{3}$	$1 - i\sqrt{3}$	$\sqrt{3} + i$	$-\sqrt{3} - i$	$-\sqrt{3} - i$	$1 - i\sqrt{3}$	$-1 + i\sqrt{3}$	$-\sqrt{3} - i$	$\sqrt{3} + i$	$1 - i\sqrt{3}$	$\sqrt{3} + i$	$-\sqrt{3} - i$	$1 - i\sqrt{3}$	$-1 + i\sqrt{3}$	$-1 + i\sqrt{3}$	$-\sqrt{3} - i$	$\sqrt{3} + i$	$-1 + i\sqrt{3}$	$1 - i\sqrt{3}$	B1 B1 B1	1.1 1.1 1.1	R ₃ and C ₃ correct Any other row or column correct All correct
			$\sqrt{3} + i$	$-\sqrt{3} - i$	$1 - i\sqrt{3}$	$-1 + i\sqrt{3}$																								
		$\sqrt{3} + i$	$-1 + i\sqrt{3}$	$1 - i\sqrt{3}$	$\sqrt{3} + i$	$-\sqrt{3} - i$																								
		$-\sqrt{3} - i$	$1 - i\sqrt{3}$	$-1 + i\sqrt{3}$	$-\sqrt{3} - i$	$\sqrt{3} + i$																								
		$1 - i\sqrt{3}$	$\sqrt{3} + i$	$-\sqrt{3} - i$	$1 - i\sqrt{3}$	$-1 + i\sqrt{3}$																								
$-1 + i\sqrt{3}$	$-\sqrt{3} - i$	$\sqrt{3} + i$	$-1 + i\sqrt{3}$	$1 - i\sqrt{3}$																										
(ii)	Closed, since no new elements in the table Since \circlearrowleft involves only complex multiplication, which is known to be Associative, this axiom holds $1 - i\sqrt{3}$ is the identity (see the Cayley table) $-1 + i\sqrt{3}$ is self-inverse and $\pm(\sqrt{3} + i)$ form an inverse-pair	B1 B1 B1 B1	2.4 2.1 1.1 2.5	Don't accept unqualified statements like "Table shows ..." Or showing, either separately or together, that $(z_1 \circlearrowleft z_2) \circlearrowleft z_3 = \frac{1}{16}(1 + i\sqrt{3})^2 z_1 z_2 z_3 = z_1 \circlearrowleft (z_2 \circlearrowleft z_3)$ All (non-identity) elements shown to have an inverse (and not just stated that this is so)																										
(iii)	$1 - i\sqrt{3}$ has order 1 $-1 + i\sqrt{3}$ has order 2 $\pm(\sqrt{3} + i)$ have order 4	B1	1.1																											
(b)	$\{1 - i\sqrt{3}, -1 + i\sqrt{3}\}$	B1 [1]	2.2a																											
(c) (i)	It has an (i.e. at least one) element of order 4	B1 [1]	2..2a	Or any equivalent, longer reason																										
(c) (ii)	$\pm(\sqrt{3} + i)$	B1	1.1	Both and no others																										

Question	Answer	Marks	AO	Guidance
		[1]		

Question	Answer	Marks	AO	Guidance
7	<p>(a)</p> $f(n) = (2^4)^n \times 2^3 + (3^3)^n \times 3$ $= 8 \times 16^n + 3 \times 27^n$ $\equiv 8 \times 5^n + 3 \times 5^n \pmod{11}$ $= 11 \times 5^n \equiv 0 \pmod{11}, \text{ as required}$	M1	3.1a	Appropriate indices work attempted
		A1	1.1	Fully correct
		M1	2.1	Reducing all terms to mod 11
		A1	2.2a	AG fully shown
		[4]		
(b)	<p>(i)</p> $10^{33} + 1 =$ a “1” followed by 32 “0”s and a final “1” Then “(Σodds) – (Σevens)” = 1 – 1 = 0 and $11 \mid 0 \Rightarrow$ divisibility by 11, as required	B1	1.1a	It is only necessary to note that there is an even number of “0”s between the first and last “1”
		B1	1.1	Proper explanation of the divisibility test
		[2]		
(ii)	Factoring out the first 11 $(10^{33} + 1)/11 =$ 15 “90”s followed by a final “91” Then “(Σodds) – (Σevens)” = $16 \times 9 - 1 = 143$ which is a multiple of 11 \Rightarrow divisibility by another factor of 11, i.e. 121	M1	3.1a	BC
		A1	1.1	
		M1	1.1	Conclusion with explanation
		A1	1.1	
<p>Alternative method</p> $10^{33} + 1 = (10^{11} + 1)(10^{22} - 10^{11} + 1)$ Showing $11 \mid 10^{11} + 1$, as in (b) (i) $(10^{11} + 1)/11 = 9\,090\,909\,091 = 11 \times 826\,446\,281$		M1		NB $(10^{11} + 1)/121 = 826\,446\,281 \Rightarrow 121 \mid 10^{33} + 1$ Must use the 11-divisibility test at least once Or via a second use of the 11-divisibility test: “(Σodds) – (Σevens)” = $5 \times 9 - 1 = 44$
A1				
		M1 A1		
		[4]		

Question	Answer	Marks	AO	Guidance
8	<p>Sequence is periodic, with period 4</p> $u_2 = a - \frac{1}{2}a^2$ $u_3 = a - \frac{\frac{1}{2}a^2}{a - \frac{1}{2}a^2} = \frac{a - a^2}{2 - a}$ <p style="text-align: center;">and $u_4 = a - \frac{\frac{1}{2}a^2(2 - a)}{a - a^2} = \frac{-a^2}{2(1 - a)}$</p> $u_5 = a - \frac{\frac{1}{2}a^2 \cdot 2(1 - a)}{-a^2} = a + (1 - a) = 1$ <p>For $a = 1$ and $a = 2$ the sequence becomes undefined Smaller period impossible (either or both of u_2, u_3) since this requires $a^2 - 2a + 2 = 0$, which has a negative discriminant</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>[7]</p>	<p>1.2</p> <p>1.1</p> <p>3.1a</p> <p>1.1</p> <p>1.1</p> <p>2.3</p> <p>2.4</p>	<p>BC (e.g.)</p> <p>Attempt at u_3, u_4 and u_5 using the given r.r.</p> <p>u_4 correct (allow unsimplified)</p> <p>u_5 correctly shown equal to 1</p> <p>Stating clearly the “illegal” values of a</p> <p>Justifying that all cases have been considered. (NB Case for u_4 is not required but may appear.)</p>

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning

Telephone: 01223 553998

Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored