OCR
Oxford Cambridge and RSA

GCE

Further Mathematics A

Y531/01: Pure Core

AS Level

Mark Scheme for June 2023

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M - A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A - Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B - Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   - When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
   - When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
     NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

   Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
   Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
   - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			$u = x + 2$	**B1**	1.1	Correct substitution stated or used	
			$(u-2)^3 = u^3 - 3\times2u^2 + 3\times2^2u - 2^3$	**M1**	1.1	Attempt to expand their $(u-2)^3$ following a linear substitution. 4 terms using $\binom{n}{r}2^r u^{n-r}$	$u^3 - 6u^2 + 12u - 8$ Binomial coefficients might not be correct or evaluated. Might be by expanding brackets.
			$(u-2)^4 = u^4 - 4\times2u^3 + 6\times2^2u^2 - 4\times2^3u + 2^4$	**M1**	1.1	Attempt to expand their $(u-2)^4$ following a linear substitution. 5 terms using $\binom{n}{r}2^r u^{n-r}$	$u^4 - 8u^3 + 24u^2 - 32u + 16$ Binomial coefficients might not be correct or evaluated. Might be by expanding brackets.
			$4(u-2)^4 - 2(u-2)^3 - 3(u-2) + 2\ (= 0)$	**M1**	1.1	Forming (LHS of) equation in u (could be using their expansions)	
			$\therefore 4(u^4 - 8u^3 + 24u^2 - 32u + 16) - 2(u^3 - 6u^2 + 12u - 8) - 3(u-2) + 2 = 0$ $\therefore\ 4u^4 - 34u^3 + 108u^2 - 155u + 88 = 0$	**A1** **[5]**	1.1	Final answer can be in terms of x Final answer needs to be an equation with coefficients simplified. ISW attempts to cancel down following correct equation	

Question			Answer	Marks	AO	Guidance	
2	(a)		$-5 + 5\lambda = 24 + 3\mu$ & $6 - 2\lambda = 1 + \mu$	B1	1.1	Forming 2 correct equations in λ and μ.	Third equation is $15 - 2\lambda = -5 - 4\mu$
			$5\lambda - 3\mu = 29$ & $6\lambda + 3\mu = 15$	M1	1.1	Attempt to solve (eg scaling one equation and adding; or rewriting to a standard form for solution BC)	If scaling or substituting method must result in correctly eliminating one variable (Other coefficients may be incorrect).
			$\lambda = 4$ & $\mu = -3$	A1	1.1	Both	
			$\lambda = 4$ & $\mu = -3$ => LHS = $15 - 2\times4 = 7$ and RHS = $-5 - 4\times-3 = 7$ = LHS so all 3 equations are satisfied so L_1 and L_2 do intersect	A1	1.1	Convincing justification but could be by finding the same **r** from both equations	
			$\begin{pmatrix}15\\-2\\7\end{pmatrix}$	A1	1.1	Condone coordinates. Can be awarded even if previous A mark not awarded (i.e. if not checked third equation)	
				[5]			
	(b)		$\begin{pmatrix}5\\-2\\-2\end{pmatrix} \times \begin{pmatrix}3\\1\\-4\end{pmatrix} = \begin{pmatrix}10\\14\\11\end{pmatrix}$	B1	3.1a	Could be BC. SOI	
			$\mathbf{r} = \begin{pmatrix}15\\-2\\7\end{pmatrix} + \nu\begin{pmatrix}10\\14\\11\end{pmatrix}$	B1FT	1.1	FT their point of intersection from **(a)** and their attempt at direction vector. SOI Condone use of λ or μ.	No need to see " r= " Must be a recognisable attempt at a vector perpendicular to both L_1 and L_2
			$\frac{x-15}{10} = \frac{y+2}{14} = \frac{z-7}{11}$	B1FT	1.1	FT their vector equation. This is for correctly turning a vector equation into a cartesian one.	Correct equation here implies the other two marks.
				[3]			

Question			Answer	Marks	AO	Guidance	
3	(a)		DR $z_1z_2 = (3 + 4i)(–5 + 12i) = –15 + 36i – 20i – 48$	**M1**	1.1	Attempt at expansion (4 terms soi) using $i^2 = –1$	DR – Need to see at least one line of expanded terms before answer
			$= –63 + 16i$	**A1** **[2]**	1.1		
	(b)		DR $\lvert z_2\rvert = (\sqrt{(-5)^2 + 12^2} = \sqrt{169}) = 13$	**B1**	1.1	Not ± unless later corrected. Allow modulus of 13 for the B1 as long as no incorrect working	Treat attempt to write z_1 or z_1z_2 in mod/arg form as MR so **B0M1A1** available
			$\tan^{-1}\left(\frac{12}{-5}\right)$	**M1**	1.1	Evidence of using trigonometry towards finding the correct angle, perhaps by finding a related angle. Treat $\tan^{-1}\left(\frac{12}{5}\right)$ as such evidence for M1 but not $\tan^{-1}\left(-\frac{5}{12}\right)$ or $\tan^{-1}\left(\frac{5}{12}\right)$ unless supported eg by a diagram or by working leading to correct answer.	
			$\therefore z_2 = 13(\cos 1.97 + i\sin 1.97)$ (3 sf)	**A1**	2.5	For argument accept awrt 1.97 only. Do not accept answers not written correctly in mod-arg form. Do not accept –1.18 or –4.32 as argument. Answer must be in radians for A1. Accept $[r, \theta]$ or r cis θ	Is 1.965587446… eg do not accept the following $13\cos 1.97 + 13i\sin 1.97$ $13(\cos 4.32 – i\sin 4.32)$ NB $z_1 = 5(\cos 0.927 + i\sin 0.927)$ $z_1z_2 = 65(\cos 2.89 + i\sin 2.89)$
				[3]			

	(c)		DR $(\arg(z_1z_2) =)\tan^{-1}\left(\frac{16}{-63}\right)$	**M1**	1.1	**Using trigonometry** to find the argument. Do not accept any other form unless supported by clear evidence (eg diagram)	This mark may be awarded if z_1z_2 was incorrect from **(a)**.
			$\arg(z_1)+\arg(z_2)=\tan^{-1}\left(\frac{4}{3}\right)+1.965587...$ $=0.927295...+1.965587...$ $=2.89288...$	**M1**	2.1	Attempt to calculate RHS using their values (either value could have been found earlier but both must be in $[0, 2\pi)$).	Could accept 0.927… as evidence of arctan(4/3)
			$\arg(z_1z_2)=-0.2487099...+\pi=2.892882...$ so they are equal	**A1** **[3]**	2.2a	Accept rounding to 3 sf or better but rounding must be correct (e.g. 0.927 + 1.96 = 2.89 would score **A0**).	Answer must be in radians for A1. If MR z_1 or z_1z_2 in part (b) then full credit available for a correct solution here.

Question			Answer	Marks	AO	Guidance	
4			**p.** $\begin{pmatrix}2\\6\\-3\end{pmatrix}=0$	**B1**	2.2a	Knowledge that perpendicularity implies that scalar product = 0, used anywhere in solution	
			$2a^2 + 6(a – 5) – 3\times26 = 0$	**M1**	3.1a	Using the scalar product to set up a quadratic equation in a	
			$\Rightarrow 2a^2 + 6a – 108 = 0$ $\Rightarrow a = 6$ or $a = –9$	**A1**	1.1	Correctly solving the equation (could be BC). Might only see $a = 6$ here.	$a^2 + 3a – 54 = (a – 6)(a + 9) = 0$ $a^2+3a-54=\left(a+\frac{3}{2}\right)^2-\frac{225}{4}$
			$a = –9$ leads to negative y component (–14) so $a = 6$ is the only solution	**A1** **[4]**	2.3	$a = –9$ or brackets $(a\text{-}6)(a+9)$ must be seen in solution and then $a = –9$ explicitly rejected with rationale	Accept "all components must be positive" for rationale. Do NOT accept "a must be positive" as sole rationale

Question			Answer	Marks	AO	Guidance	
5			DR $\alpha + \beta = \frac{3}{5}$	**B1**	1.1		
			$\alpha\beta = \frac{12}{5}$	**B1**	1.1		
			$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$	**M1**	3.1a	Rewrite the expression in terms of the standard symmetric functions	Need to see $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ oe No need to see $\alpha^2\beta^2 = (\alpha\beta)^2$
			$= \frac{\left(\frac{3}{5}\right)^2 - 2\times\frac{12}{5}}{\left(\frac{12}{5}\right)^2} = -\frac{37}{48}$	**A1**	1.1	cao Accept eg $-0.77083\dot{3}$ but not rounded, incorrect or incomplete decimal form.	SC B1 for correct answer if B0B0M0 Accept any equivalent fraction
				[4]			

Question			Answer	Marks	AO	Guidance	
6			$n = 0$, $4\times1 + 66 = 70 = 5\times14$ is divisible by 14	**B1**	2.1	Basis case. Must explicitly see 70 and state, or show, divisibility.	If $n = 1$ (leading to $98 = 7\times14$) used and not corrected then allow this mark but withhold final mark.
			Assume true for $n = k$ ie that $4\times8^k + 66$ is divisible by 14 oe	**M1**	2.1	Statement of inductive hypothesis. Allow "$= 14p$" without further qualification	
			Considering $4\times8^{k+1} + 66$ and rewriting it as $4\times8\times8^k + 66$ or $32\times8^k + 66$ oe	**M1**	1.1	Uses law of indices correctly to obtain expression in terms of 8^k and no other exponential term	Could consider f(k+1)—f(k) or similar
			$= 8(14p - 66) + 66$ from inductive hypothesis	**M1**	1.1	Uses inductive hypothesis properly. Do not allow if eg 32^k (ie law of indices must have been correctly used)	Allow for substitution of their $n=k$ case into their $n=k+1$ case
			$= 112p - 462$ or $14\times8p - 14\times33$ oe $=14(8p - 33)$ which is divisible by 14	**A1**	2.2a	Simplification with sufficient working to establish and state divisibility for $k + 1$.	Must either show 14× explicitly in each term or 14 is a factor
			So true for $n = k \Rightarrow$ true for $n = k + 1$. But true for $n = 0$. So true for all integers $n \geq 0$	**A1**	2.4	Clear conclusion for induction process. See note for basis case above. Do not allow "true for all positive integers". Need to see if true for n=k, then true for n=k+1 i.e. k and k+1th case linked to n. Could see Proposition notation ($P_k \Longrightarrow P_{k+1}$)	A formal proof by induction, with no gaps in logic, is required for full marks. Full marks can be gained by using induction to prove that $2\times8^n + 33$ is divisible by 7 for all $n \geq 0$ and observing that $4\times8^n + 66 = 2(2\times8^n + 33)$ is divisible by 2 and $2\times8^n + 33$ and hence by both 2 and 7 and hence by 14
				[6]			

Question	Answer	Marks	AO	Guidance	
7	DR det**A** = $a(a(a-1)-4\times(-13))-(-6)((a+9)(a-1)-0)+(a-3)((a+9)(-13)-0)$	**M1**	3.1a	Attempt to expand the determinant. If using standard method must see at least two terms, at least one of which comprises ± a number multiplied by the residual determinant. eg $a(a(a-1)-4\times(-13))-(a+9)((-6)(a-1)-(a-3)(-13))$	This mark can be awarded with a = 2 substituted (ie attempt to find $\begin{vmatrix} 2 & -6 & -1 \\ 11 & 2 & 4 \\ 0 & -13 & 1 \end{vmatrix}$) but working must be shown. eg $2(2+52)-11(-6-13)$ (= 317) Allow other correct methods.
	$= a^3 - 8a^2 + 22a + 297$	**A1**	1.1		
	$a^3 - 8a^2 + 22a + 297 = 2^3 - 8\times2^2 + 22\times2 + 297$ (or $a^3 - 8a^2 + 22a + 297 = 317$)	**M1**	2.2a	Setting up equation equating their determinant to their specific determinant value for a = 2	
	$a^3 - 8a^2 + 22a - 20 = 0$ $a^2(a-2) - 6a(a-2) + 10(a-2) = 0$ $a^2 - 6a + 10 = 0$	**M1**	1.1	Rearranging to = 0 and use of factor theorem to derive a quadratic equation in a by dividing by $(a-2)$	Need some evidence of how to solve cubic (Not just cubic written and then roots BC). Need to see =0 on one side of cubic (but it might disappear after this, e.g. when dividing by (a-2))
	$(a-3)^2 - 9 + 10 = 0$ $(a-3)^2 = -1$ $a - 3 = \pm i$	**M1**	1.1	Attempt to solve quadratic involving $\sqrt{-1} = i$ oe	$\frac{6 \pm \sqrt{6^2 - 4\times1\times10}}{2\times1} = \frac{6 \pm \sqrt{-4}}{2} = \frac{6 \pm 2i}{2}$
	(a =) $3 \pm i$ (and a = 2)	**A1**	1.1	Both. No need to mention a = 2 Don't need to see "a=" explicitly	SC – if third and/or fourth method mark not awarded then allow SC B1 for sight of $3 \pm i$.
		[6]			

Question			Answer	Marks	AO	Guidance	
8	(a)		$\omega = a + ib$ oe	M1	3.1a	Writing ω in a form which allows 2 equations to be found oe (eg taking Re and Im of both sides)	
			$a + 2 = 3a$ $b + 7 = -3b - 1$	M1	1.1	Equating real and imaginary parts. Aef	This might happen after some simplification.
			$a = 1$ or $b = -2$	A1	1.1		
			$(\omega =)\ 1 - 2i$	A1 [4]	1.1	Need to see answer as a complex number.	Ignore presence of ω^*=1+2i as long as 1-2i identified as ω.
	(b)		If z is purely imaginary, so $z = ki$ for some real k, then $z^* = -ki = -z$ as required	B1	2.1	<=. This could, with care, be included in the => proof.	
			If $z = r + si$ (r, s real) then $z^* = r - si$ so $z = -z^*$ => $r + si = -(r - si) = -r + si$ => $r = 0$ (so $s \neq 0$ since z is non-zero) so z is purely imaginary	B1 [2]	2.1	=>. z being non-zero does not have to be rigorously dealt with.	Could instead consider if z is not purely imaginary and show this means $z^* \neq -z$
	(c)	(i)	Reflection in the real axis	B1 [1]	1.2	Must be real, rather than x, axis.	If mention of real axis, can ignore x
		(ii)	$z = z^*$ means that A and B are coincident so A is an invariant point so A must lie on the mirror line, which is the real axis, so A must represent a purely real number so z is purely real.	B1	2.4	=>. Could, with care, be included in the <= proof.	Needs to be a geometric explanation.
			If z is purely real then A lies on the real axis so it is invariant under a reflection in the real axis so the conjugate z^* is also represented by the same point so $z = z^*$	B1 [2]	2.4	<=.	

Question			Answer	Marks	AO	Guidance	
9	(a)		$\begin{pmatrix} a & 0 & -b \\ 0 & 1 & 0 \\ b & 0 & a \end{pmatrix}\begin{pmatrix} a & 0 & -b \\ 0 & 1 & 0 \\ b & 0 & a \end{pmatrix}$ $= \begin{pmatrix} a\times a + -b\times b & 0 & -ab - ab \\ 0 & 0+1\times 1+0 & 0 \\ ab+ab & 0 & -b\times b + a\times a \end{pmatrix}$	M1	1.1	Some indication of knowledge of how to multiply 3 by 3 matrices	Can be implied by 3 non-zero correct terms
			$= \begin{pmatrix} a^2-b^2 & 0 & -2ab \\ 0 & 1 & 0 \\ 2ab & 0 & a^2-b^2 \end{pmatrix}$	A1	1.1	Final solutions must have all terms simplified.	
				[2]			
	(b)		$ab = \frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)\frac{\sqrt{2}}{4}\left(\sqrt{3}-1\right) = \frac{2}{16}(3-1) = \frac{1}{4}$ or $a^2-b^2 = (a-b)(a+b) = \frac{2\sqrt{2}}{4}\times\frac{2\sqrt{2}\sqrt{3}}{4}$ $= \frac{1}{2}\sqrt{3}$	B1	3.1a	Explicitly finding an expression for either ab (or $2ab$) or $a^2 - b^2$	
			$R^2 = \begin{pmatrix} \frac{1}{2}\sqrt{3} & 0 & -2\times\frac{1}{4} \\ 0 & 1 & 0 \\ 2\times\frac{1}{4} & 0 & \frac{1}{2}\sqrt{3} \end{pmatrix}$ $= \begin{pmatrix} \frac{1}{2}\sqrt{3} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2}\times 2 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2}\sqrt{3} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \sqrt{3} & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & \sqrt{3} \end{pmatrix}$ so $k = ½$	B1	2.2a	AG. Explicitly finding the expression for the other substituting into expression for $\mathbf{R}^2$ (or carrying out the matrix multiplication again). k can be embedded.	If k embedded need to see $\frac{1}{2}\times 2$ or $\frac{2}{2}$.
				[2]			

	(c)		$\mathbf{R}^4 = \begin{pmatrix} \frac{1}{2}\sqrt{3} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2}\sqrt{3} \end{pmatrix}\begin{pmatrix} \frac{1}{2}\sqrt{3} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2}\sqrt{3} \end{pmatrix}$ $= \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2}\sqrt{3} \\ 0 & 1 & 0 \\ \frac{1}{2}\sqrt{3} & 0 & \frac{1}{2} \end{pmatrix}$ $\mathbf{R}^6 = \begin{pmatrix} \frac{1}{2}\sqrt{3} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2}\sqrt{3} \end{pmatrix}\begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2}\sqrt{3} \\ 0 & 1 & 0 \\ \frac{1}{2}\sqrt{3} & 0 & \frac{1}{2} \end{pmatrix}$ $= \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$			For reference – does not need to be seen in working.	SC – if answers left in terms of k then award B1 if one or two correct and B2 if all three correct
				B1	1.1	For correct $\mathbf{R}^6$	By calculator expected, so intermediate steps might not be shown.
			$\mathbf{R}^{12} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ $= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	**B1**	1.1	For correct $\mathbf{R}^{12}$	

			$\mathbf{R}^{24} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ $= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	**B1** **[3]**	1.1	For correct $\mathbf{R}^{24}$	
	(d)		Rotation	**B1**	3.1a		
			(360°/24 =) 15°	**B1**	2.2a	Also allow 345° (Rotation in opposite sense)	Also allow in radians $\frac{\pi}{12}$
			Clockwise about the *y*-axis.	**B1** **[3]**	3.2a	Both sense and axis must be correct. If correct transformation is combined with an incorrect one (such as correct rotation combined with a reflection) then maximum mark is B2.	Could be a rotation of 345° anticlockwise about y-axis

Need to get in touch?

If you ever have any questions about OCR qualifications or services (including administration, logistics and teaching) please feel free to get in touch with our customer support centre.

Call us on

01223 553998

Alternatively, you can email us on

support@ocr.org.uk

For more information visit

ocr.org.uk/qualifications/resource-finder

ocr.org.uk

Twitter/ocrexams

/ocrexams

/company/ocr

/ocrexams

CAMBRIDGE
UNIVERSITY PRESS & ASSESSMENT

OCR is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

OCR operates academic and vocational qualifications regulated by Ofqual, Qualifications Wales and CCEA as listed in their qualifications registers including A Levels, GCSEs, Cambridge Technicals and Cambridge Nationals.

OCR provides resources to help you deliver our qualifications. These resources do not represent any particular teaching method we expect you to use. We update our resources regularly and aim to make sure content is accurate but please check the OCR website so that you have the most up-to-date version. OCR cannot be held responsible for any errors or omissions in these resources.

Though we make every effort to check our resources, there may be contradictions between published support and the specification, so it is important that you always use information in the latest specification. We indicate any specification changes within the document itself, change the version number and provide a summary of the changes. If you do notice a discrepancy between the specification and a resource, please contact us.

Whether you already offer OCR qualifications, are new to OCR or are thinking about switching, you can request more information using our Expression of Interest form.

Please get in touch if you want to discuss the accessibility of resources we offer to support you in delivering our qualifications.