AS
FURTHER MATHEMATICS
7366/2S

Paper 2 Statistics

Mark scheme

June 2023

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student’s best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.1b	B1	13
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	2
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Uses correct formula for the mean of a discrete random variable to calculate E(X) or E($5X$) or calculates values of $5x - 7$ for each value of x	1.1a	M1	E(X) = –4 × 0.2 + 3 × 0.7 + 8 × 0.1 E(X) = 2.1 E($5X$ – 7) = 5E(X) – 7 E($5X$ – 7) = 5 × 2.1 – 7 E($5X$ – 7) = 3.5
	States or uses correct formula for E($5X$ – 7) **Not** implied by sight of E($5X$ – 7) = 3.5 following a correct value of E(X) or E($5X$)	1.1a	M1	
	Completes a reasoned argument by giving a calculation that obtains the given value of E($5X$ – 7)	2.1	R1	
	Question total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
4	Uses Binomial model $X \sim \mathrm{B}(30,\ 0.35)$ to calculate any probability	3.4	M1	$X \sim \mathrm{B}(30,\ 0.35)$ $\mathrm{P}(X \leq 6) = 0.05857 > 0.025$ $\mathrm{P}(X \leq 5) = 0.02326 < 0.025$ $\mathrm{P}(X \leq 15) = 0.96992$ $\mathrm{P}(X \geq 16) = 1 - \mathrm{P}(X \leq 15)$ $\mathrm{P}(X \geq 16) = 1 - 0.96992$ $\mathrm{P}(X \geq 16) = 0.03008 > 0.025$ $\mathrm{P}(X \leq 16) = 0.98764$ $\mathrm{P}(X \geq 17) = 1 - \mathrm{P}(X \leq 16)$ $\mathrm{P}(X \geq 17) = 1 - 0.98764$ $\mathrm{P}(X \geq 17) = 0.01236 < 0.025$ Probability of Type I error = 0.02326 + 0.01236 = 0.03562 = 0.0356 (4 dp)
	Obtains the correct value or calculation of $\mathrm{P}(X \leq 5)$, **AWRT** 0.023, and the correct value or calculation of $\mathrm{P}(X \geq 17)$, **AWRT** 0.012	1.1b	A1	
	Adds the probability of their lower tail to the probability of their upper tail	1.1a	M1	
	Completes a reasoned argument to obtain the given value or a value that rounds to the given answer of the probability of Type I error. The correct values, rounded or truncated, of $\mathrm{P}(X \leq 5)$, $\mathrm{P}(X \leq 6)$, $\mathrm{P}(X \geq 16)$ and $\mathrm{P}(X \geq 17)$ or $\mathrm{P}(X \leq 5)$, $\mathrm{P}(X \leq 6)$, $\mathrm{P}(X \leq 15)$ and $\mathrm{P}(X \leq 16)$ all need to be seen. Probabilities in the sum of the lower and upper tails need to be given correct to at least 5 decimal places.	2.1	R1	
	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Obtains correct value of $\overline{x}$ **PI**	1.1b	B1	$\overline{x} = \frac{35522}{100} = 355.22$ $s^2 = \frac{1}{99}\left(32902257 - \frac{35522^2}{100}\right)$ $= 204890.2238$ $z = 2.1701$ $355.22 \pm 2.1701 \times \sqrt{\frac{204\ 890.2238}{100}}$ (257, 453)
	Obtains correct value of s^2 or s **AWRT** $s^2 = 204\,890$ or $s = 453$ **PI**	1.1b	B1	
	Obtains correct z value **AWRT** 2.17 or correct t value **AWRT** 2.20 **PI**	1.1b	B1	
	Uses formula for upper or lower limit of a confidence interval using their values. **PI**	1.1a	M1	
	Obtains correct confidence interval **AWRT** 257 and 453 or 256 and 455 if t value used **CSO**	1.1b	A1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Infers that Rebekah's statement or Mike's claim is supported by the confidence interval by comparing the proposed population mean with their confidence interval provided that 267 lies within it Condone use of "it" for 267	2.2b	E1F	Rebekah is correct as 267 is within the confidence interval
	Subtotal		**1**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Obtains the correct probability **AWRT** 0.293	1.1b	B1	0.293
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Selects and uses a Poisson model with $\lambda = \frac{65}{24}$ to find P(number of motor claims = 2) or P(number of motor claims $\leq$ 2) **PI**	3.3	M1	$X \sim \text{Po}\left(\frac{65}{24}\right)$ $\text{P}(X = 2) = 0.244$
	Obtains the correct probability **AWRT** 0.244	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)(i)	Selects Poisson model and attempts the square root of their λ or calculates 88 **PI**	3.3	M1	$\lambda = 65 + 23 = 88$ $Y \sim \text{Po}\ (88)$ Standard deviation = $\sqrt{88}$ = 9.38
	Obtains the correct standard deviation **AWRT** 9.38	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)(ii)	Uses Poisson model with $\lambda = 88$ or their λ from **(c)(i)** to correctly find P($Y \leq 90$) **AWRT** 0.61, P($Y \geq 91$) **AWRT** 0.39, P($Y \leq 89$) **AWRT** 0.57 or P($Y \geq 90$) **AWRT** 0.43 **PI**	3.4	M1	Y ~ Po (88) P($Y > 90$) = 1 – P($Y \leq 90$) P($Y > 90$) = 1 – 0.611… P($Y > 90$) = 0.389
	Obtains the correct probability **AWRT** 0.389	1.1b	A1	
	Subtotal		**2**	

	Question total		**7**	

<table>
<tr><th>Q</th><th>Marking instructions</th><th>AO</th><th>Marks</th><th>Typical solution</th></tr>
<tr><td rowspan="7">7(a)</td><td>States both hypotheses using correct language. Variables need to be stated in at least the null hypothesis</td><td>2.5</td><td>B1</td><td rowspan="7">H_0: There is no association between the show a customer attends and whether they enjoyed the show
H_1: There is an association between the show a customer attends and whether they enjoyed the show

Expected Frequencies
<table><tr><th></th><th>MS</th><th>AS</th><th>ES</th></tr><tr><td>En</td><td>56.55</td><td>81.90</td><td>186.55</td></tr><tr><td>NEn</td><td>30.45</td><td>44.10</td><td>100.45</td></tr></table>
$\sum\frac{(O-E)^2}{E}$
<table><tr><th></th><th>MS</th><th>AS</th><th>ES</th></tr><tr><td>En</td><td>0.5252</td><td>1.0111</td><td>1.1348</td></tr><tr><td>NEn</td><td>0.9755</td><td>1.8778</td><td>2.1075</td></tr></table>
$\sum\frac{(O-E)^2}{E} = 7.6$
χ^2 cv for 2 df $=$ 7.378
7.6 $>$ 7.378
Reject H_0
Sufficient evidence to suggest that there is an association between the show a customer attends and whether they enjoyed the show</td></tr>
<tr><td>Translate situation into expected contingency table for χ^2 model</td><td>3.3</td><td>M1</td></tr>
<tr><td>Uses χ^2 model to calculate test statistic</td><td>3.4</td><td>M1</td></tr>
<tr><td>Obtains correct value of
$\sum\frac{(O-E)^2}{E}$
AWRT 7.6</td><td>1.1b</td><td>A1</td></tr>
<tr><td>Obtains correct critical value for the test
AWRT 7.4
or
corresponding probability of test statistic
AWRT 0.022</td><td>1.1b</td><td>B1</td></tr>
<tr><td>Evaluates χ^2-test statistic by correctly comparing their critical value with their test statistic or the probability with 0.025</td><td>3.5a</td><td>R1</td></tr>
<tr><td>Infers H_0 rejected
FT their comparison using the χ^2 model.</td><td>2.2b</td><td>E1F</td></tr>
</table>

	States the conclusion in context (The conclusion must not be definite.) **FT** their incorrect acceptance of H_0 if stated or 'their' comparison if not	3.2a	E1F	
	Subtotal		**8**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Explains reasoning by considering $(O-E)$ or $\dfrac{(O-E)^2}{E}$ to identify largest source of association The value of $(O-E)$ or $\dfrac{(O-E)^2}{E}$ does not need to be seen provided that it is identified as the largest	2.4	E1	Largest sources of association evening show/did not enjoy $\dfrac{(O-E)^2}{E} = 2.1\ldots$ People who attend the evening show did not enjoy the show more often than expected
	Interprets the main source of association in context by stating either that People who attend the evening show did not enjoy the show more often than expected or People who attend the evening show enjoyed the show less often than expected **OE** Needs to be a comparison, not just listing observed and expected frequencies	3.2a	E1	
	Subtotal		**2**	

	Question total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
8	Uses one of the points shown on the graph and gradient $\frac{4}{3}$ to find an equation of the straight-line section of the probability density function **PI**	3.1a	M1	$m=\frac{\frac{5}{3}-1}{\frac{3}{2}-1}=\frac{4}{3}$ $f(x)-1=\frac{4}{3}(x-1)$ for $1< x\le\frac{3}{2}$ $f(x)=\frac{4}{3}x-\frac{1}{3}$ for $1< x\le\frac{3}{2}$ $E(X)=\int_0^1 x^3\,dx+\int_1^{\frac{3}{2}} x\left(\frac{4}{3}x-\frac{1}{3}\right)dx$ $=\frac{1}{4}+\frac{61}{72}$ $=\frac{79}{72}$ $E(X^2)=\int_0^1 x^4\,dx+\int_1^{\frac{3}{2}} x^2\left(\frac{4}{3}x-\frac{1}{3}\right)dx$ $=\frac{1}{5}+\frac{157}{144}$ $=\frac{929}{720}$ $\text{Var}(X) = E(X^2) - (E(X))^2$ $=\frac{929}{720}-\left(\frac{79}{72}\right)^2$ $= 0.08638$ $= 0.0864$ (3sf)
	Obtains a correct expression $\frac{4}{3}x-\frac{1}{3}$ for the straight-line section of the probability density function	1.1b	A1	
	Uses the formula for $E(g(X))$ to obtain a correct expression using their equation of the straight-line section for $E(X)$ or $E(X^2)$ **PI** Condone missing brackets	1.1a	M1	
	Obtain correct expressions for $E(X)$ and $E(X^2)$ **PI** **FT** their equation of the straight-line section	1.1b	A1F	
	Obtains the correct value of $E(X)$ **AWRT** 1.10 **PI** by correct calculation substituted into calculation to find $\text{Var}(X)$	1.1b	A1	
	Obtains the correct value of $E(X)^2$ **AWRT** 1.29 **PI** by correct calculation substituted into calculation to find $\text{Var}(X)$	1.1b	A1	
	Uses the formula for the variance to obtain a calculation to find $\text{Var}(X)$	1.1a	M1	

	Completes a reasoned argument to obtain the given value of Var(X) A more accurate value needs to be seen before rounding, **AWRT** 0.08638	2.1	R1	
	Question total		**8**	

	Question Paper total		**40**	