GCE A LEVEL MARKING SCHEME

SUMMER 2019

A LEVEL (NEW)
FURTHER MATHEMATICS
UNIT 6 FURTHER MECHANICS B
1305U60-1

INTRODUCTION

This marking scheme was used by WJEC for the 2019 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE FURTHER MATHEMATICS

A2 UNIT 6 FURTHER MECHANICS B

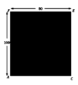
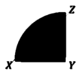
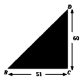
SUMMER 2019 MARK SCHEME

Q1	Solution	Mark	Notes
(a)	(i) $980 = 80 + 0 \cdot 1v^2$ $v^2 = 9000$ $v = 30\sqrt{10} = 94 \cdot 868 \ldots \ (\text{ms}^{-1})$	M1 A1	At max. speed, $a = 0$ cao
	(ii) Apply N2L, $980 - (80 + 0 \cdot 1v^2) = 360a$ $360v\frac{\mathrm{d}v}{\mathrm{d}x} = 900 - 0 \cdot 1v^2$ $3600v\frac{\mathrm{d}v}{\mathrm{d}x} = 9000 - v^2$	M1 A1 A1 **[5]**	Dim. correct, Fully correct equation (or $980\,000 - 80\,000 - 100v^2 = 360\,000a$) Convincing
(b)	$3600\int \frac{v}{9000-v^2}\,\mathrm{d}v = \int \mathrm{d}x$	M1	Separate variables **including** attempt to integrate
	$-\frac{3600}{2}\ln\lvert 9000 - v^2\rvert = x \quad (+C)$	A1 A1	$\ln\lvert 9000 - v^2\rvert$ Everything correct
	When $x = 0, v = 0$	m1	Use of initial conditions
	$-\frac{3600}{2}\ln\lvert 9000\rvert = C$	A1	cao
	$x = 1800\ln\lvert 9000\rvert - 1800\ln\lvert 9000 - v^2\rvert$ $\frac{x}{1800} = \ln\left\lvert\frac{9000}{9000 - v^2}\right\rvert$		
	$\frac{9000}{9000 - v^2} = e^{\frac{x}{1800}}$	m1	inversion
	$9000 - v^2 = 9000e^{-\frac{x}{1800}}$		
	$v^2 = 9000\left(1 - e^{-\frac{x}{1800}}\right)$	A1 **[7]**	oe, cao
(c)	When $v = 85$, $x = 1800\ln\left\lvert\frac{9000}{9000 - (85)^2}\right\rvert$	M1	Used in expression for v^2 or x Accept use of inequalities.
	$x = 2922.1634 \ldots$ $x = 2923 \text{ m}$ or $x = 2 \cdot 923 \text{ km}$	A1 **[2]**	cao

(d)	Appropriate explanation, e.g. • When $v = 30\sqrt{10}$ (or $v^2 = 9000$) we get division by zero in expression for x • $v = 30\sqrt{10}$ is a limiting value	E1 **[1]**	
	Total for Question 1	**15**	

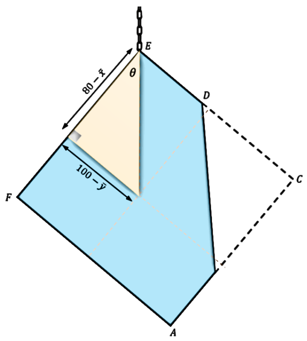
Q2	Solution	Mark	Notes

(a)

Shape	Area/mass	Distance from AF	Distance from AC	Mark	Notes
	80×100 $(= 8000)$	40	50	B1	
	$\frac{\pi(24)^2}{4}$ (144π)	$29 - \frac{32}{\pi}$	$60 + \frac{32}{\pi}$	B1	cao for 1st three B1's
	$\frac{51 \times 60}{2}$ $(= 1530)$	$29 + 34$	20	B1	
●	$0 \cdot 5 \times 1530$ $(= 765)$	29	60	B1	FT their triangle
Sign	$7\,235 + 144\pi$	$\bar{x}$	$\bar{y}$	B1	

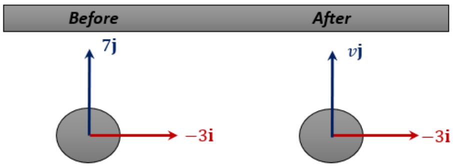
Solution	Mark	Notes
(i) Moments about AF	M1	Masses and moments consistent
$(7\,235 + 144\pi)\bar{x} = (8000)(40) + (144\pi)\left(29 - \frac{32}{\pi}\right)$ $-(1530)(63) + (765)(29)$	A1	
$\bar{x} = 33 \cdot 08 \ldots \quad \text{(cm)}$	A1	cao
(ii) Moments about AC	M1	Masses and moments consistent
$(7\,235 + 144\pi)\bar{y} = (8000)(50) + (144\pi)\left(60 + \frac{32}{\pi}\right)$ $-(1530)(20) + (765)(60)$	A1	
$\bar{y} = 58.15 \ldots \quad \text{(cm)}$	A1	cao
	[11]	

(b)

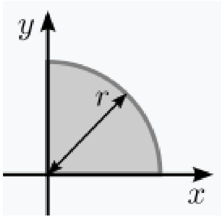


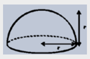
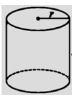
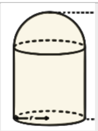
Solution	Mark	Notes
If hanging in equilibrium, vertical passes through centre of mass.	M1	Correct triangle FT $\bar{x}$ and $\bar{y}$ from (a)
$\theta = \tan^{-1}\left(\frac{100 - \bar{y}}{80 - \bar{x}}\right)$	A1	FT $\bar{x}$ and $\bar{y}$ from (a)
$\theta = 41 \cdot 729 \ldots^{\circ}$	A1	FT $\bar{x}$ and $\bar{y}$ from (a)
	[3]	
Total for Question 2	**14**	

Q3	Solution	Mark	Notes
(a)	If e is the extension in equilibrium position $mg = \frac{14e}{l}$	M1	Use of Hooke's Law
	extended length, $e = \frac{mgl}{14}$	A1	
		[2]	
(b)	(i) $T = \frac{14(e+x)}{l}$	M1	
	$= \frac{14e}{l} + \frac{14x}{l}$		
	$= mg + \frac{14x}{l}$	A1	oe, e eliminated
	(ii) Apply N2L	M1	Dim. correct. mg and T opposing
	$mg - T = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$	A1	
	$mg - \left(mg + \frac{14x}{l}\right) = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$		
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{14}{ml}x$	A1	Convincing
	(iii) Maximum distance is $e = \frac{mgl}{14}$		FT distance in (a)
	AND String would become slack.	E1	Distance and reason must be seen.
		[6]	
(c)	(i) $\omega = \sqrt{\frac{14}{0 \cdot 5 \times 0 \cdot 7}} = \sqrt{40} = 2\sqrt{10}$	B1	
	$a = 0 \cdot 2$	B1	
	Maximum speed $= a\omega$	M1	Used with their a, ω
	$= 0 \cdot 2 \times 2\sqrt{10}$ $= \frac{2}{5}\sqrt{10}$ or $1 \cdot 26 \ldots$	A1	cao
	(ii) Using $x = a\cos\omega t$ with $\omega = \sqrt{40}$ and $0 \cdot 15$		
	$0 \cdot 15 = 0 \cdot 2 \cos\sqrt{40}t$ $\sqrt{40}t = 0 \cdot 7227 \ldots$	M1	FT a, ω
	$t = 0 \cdot 11(4 \ldots)$ (s)	A1	cao
		[6]	
	Total for Question 3	**14**	

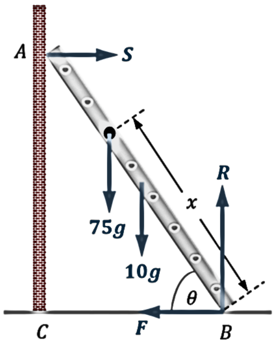
Q4	Solution	Mark	Notes
(a)	Conservation of momentum, $m\mathbf{u}_A + 0 = m\,\mathbf{v}_A + m\,\mathbf{v}_B$ $m(-\mathbf{i} + 8\mathbf{j}) + 0 = m(2\mathbf{i} + \mathbf{j}) + m\mathbf{v}_B$ $\mathbf{v}_B = (-3\mathbf{i} + 7\mathbf{j})$ (ms^{-1})	M1 A1 A1 **[3]**	Used Convincing
(b)	 Restitution parallel to **j** $v = -\frac{5}{7}(7) = -5$ $\mathbf{v} = -3\mathbf{i} - 5\mathbf{j}$	M1 A1 A1 **[3]**	Used Velocity parallel to the cushion remains unchanged, i.e. $-3\mathbf{i}$
(c)	Impulse, $\mathbf{I}$ = change in momentum $\mathbf{I} = m(-3\mathbf{i} - 5\mathbf{j}) - \mathrm{m}(-3\mathbf{i} + 7\mathbf{j})$ $\mathbf{I} = -12m\mathbf{j}$ (Ns) $\lvert\mathbf{I}\rvert = 12m$ Ns	M1 A1 A1 **[3]**	Used **Units must be included,** cao
(d)	(i) $\mathbf{r} = \mathbf{p} + \mathbf{v}t$ $\mathbf{r} = (x\mathbf{i} + 1\cdot75\mathbf{j}) + (-3\mathbf{i} - 5\mathbf{j})t$ Let $\mathbf{r}_{\text{pocket}} = 1\cdot75\mathbf{i}$ and compare **j** coefficients to get $t = 0\cdot35$ (s) (ii) Comparing **i** coefficients $x = 2.8$ (m) Alternative solution (i) Parallel to y-axis, time $= \frac{\text{distance}}{\text{speed}} = \frac{1\cdot75}{5}$ $= 0\cdot35$ (s) (ii) Parallel to the x-axis, dist. = speed × time $= 3 \times 0\cdot35$ $x = 1\cdot05 + 1.75$ $x = 2.8$ (m)	 M1 A1 M1 A1 **[4]** (M1) (A1) (M1) (A1) **([4])**	 Using $t = 0$ for instant of impact with table cushion and attempt at comparing both coefficients must be made

(e)	Any sensible refinement, e.g. • Include friction between table and ball. • Consider air resistance, Any valid explanation, e.g. • Ball will take longer to enter pocket • Ball may stop before entering the pocket	B1 E1 **[2]**	
Total for Question 4		**15**	

Q5	Solution	Mark	Notes
(a)	 $y^2 = r^2 - x^2$		
	$(V\bar{x} =)\ \pi \int_0^r xy^2\,\mathrm{d}x$	M1	Used with $y^2 = ax^2 + b$
	$(V\bar{x}) = \pi \int_0^r x(r^2 - x^2)\,\mathrm{d}x$	A1	All correct, $y^2 = r^2 - x^2$
	$(V\bar{x}) = \pi \left[\frac{r^2x^2}{2} - \frac{x^4}{4}\right]_0^r$		
	$(V\bar{x}) = \frac{1}{4}\pi r^4$	A1	
	Using $V = \frac{1}{2}\times\frac{4}{3}\pi r^3$ and dividing to get		
	$\bar{x} = \frac{3}{8}r$	A1	convincing
		[4]	
	<u>Alternative solution (applied as above)</u>		
	Equivalently, allow for		
	$(V\bar{y}) = \pi \int_0^r x^2y\,\mathrm{d}y$		
	$\bar{y} = \frac{3}{8}r$		

(b) Shape	Mass	Distance from plane face	Mark
	$\frac{1}{2}\times\frac{4}{3}\pi r^3\times\frac{3}{2}\rho$ $(= \pi r^3\rho)$	$2r + \frac{3r}{8}$ $\left(= \frac{19}{8}r\right)$	B1
	$\pi r^2 \times 2r \times \rho$ $(= 2\pi r^3\rho)$	r	B1
	$\pi r^3\rho + 2\pi r^3\rho$ $(= 3\pi r^3\rho)$	$\bar{h}$	B1

	Solution	Mark	Notes
	Taking moments	M1	No extra/missing terms
	$3\pi r^3\rho \times \bar{h} = \pi r^3\rho \times \frac{19}{8}r + 2\pi r^3\rho \times r$	A1	FT table provided dim. correct
	$\bar{h} = \frac{35}{24}r$	A1	cao
		[6]	
	Total for Question 5	**10**	

Q6	Solution	Mark	Notes
(a)			
	Resolve horizontally $S = F$	B1	si
	Moments about B	M1	Dim. correct equation with 3 terms
	$10g \times 2\cdot 5\cos\theta + 75g \times x\cos\theta = S \times 5\sin\theta$ $S \times 5\sin\theta = 25g\cos\theta(1+3x)$ $S = \dfrac{25g\cos\theta(1+3x)}{5\sin\theta}$	A2	-1 each error
	$F = 5g\cot\theta(1+3x)$	A1	Convincing
		[5]	
(b)	Use of $F = 5g\cot\theta\,(1+3x)$ with $x = 5$ and $\tan\theta = 4$ $F = 5g \times \frac{1}{4} \times 16$	M1	
	$F = 20g = 196$ (N)	A1	Accept any form, cao
	Resolve vertically $R = 10g + 75g = 85g$ $(= 833)$	M1 A1	Dim. correct equation – working may be seen in (a) $F = \mu R$ si
	Use of $F \le \mu R$	M1	
	$\mu \ge \frac{F}{R} = \frac{20g}{85g} = 0\cdot 235\ldots$	A1	cao
		[6]	
(c)	Woman modelled as a particle Ladder is a **rigid** rod	E1	
		[1]	
	Total for Question 6	**12**	